The General Linear Model

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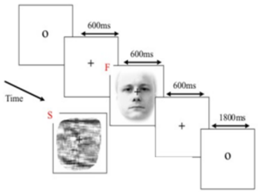
Data example

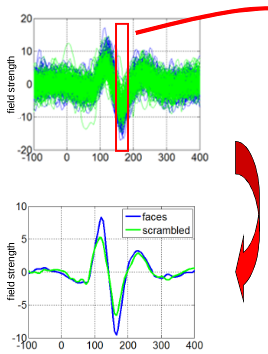
• Random presentation of ***Famous***, ***Unfamiliar*** and ***Scrambled*** faces.

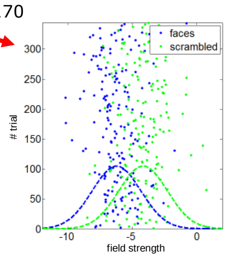


Question:
Is there a difference between the ERF of ‘*faces*’ and ‘*scrambled faces*’?

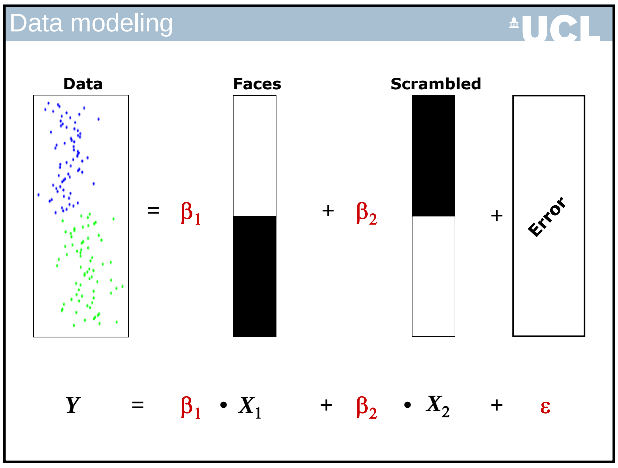
Evoked field example

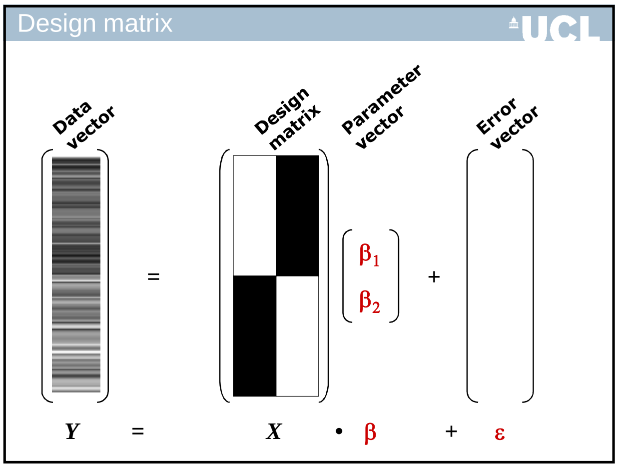
Focus on N170

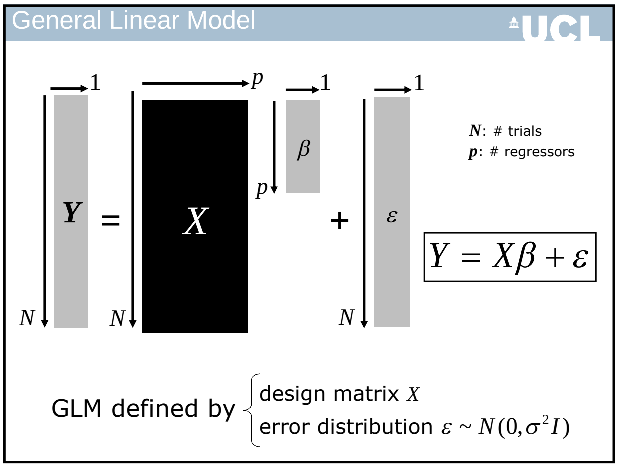




$$t = \frac{\mu_1 - \mu_2}{\sqrt{\hat{\sigma}^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

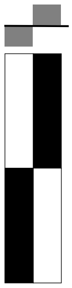

Data modeling
UCL
Data
Faces
Scrambled
Error
= β1 + β2 +
Y = β1 • X1 + β2 • X2 + ε


Design matrix
UCL
Data vector
Design matrix
Parameter vector
Error vector
β1
β2
= +
Y = X • β + ε


General Linear Model
UCL
1
p
1
1
Y = X + ε
β
p
N N N
N: # trials
p: # regressors
Y = Xβ + ε
GLM defined by
design matrix X
error distribution ε ~ N(0,σ²I)

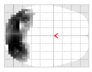
Contrast : specifies linear combination of parameter vector: $c^T \beta$

c^T = -1 +1



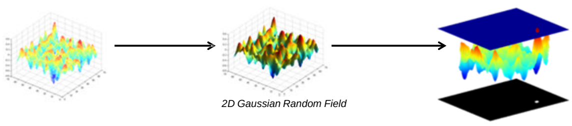
ERP: Famous < Unfamiliar?
=
$\hat{\beta}_1 < \hat{\beta}_2$?
$-1\hat{\beta}_1 + 1\hat{\beta}_2 > 0$?
Test H$_0$: $c^T \hat{\beta} > 0$?

t-statistic map



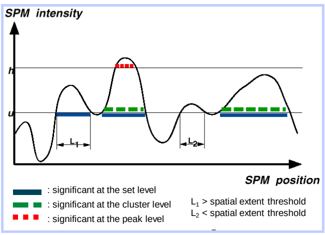
$t = \frac{\textit{contrast of estimated parameters}}{\sqrt{\text{variance estimate}}}$ → $t = \frac{c^T \hat{\beta}}{\sqrt{s^2 c^T (X^T X)^+ c}}$

Peak, cluster and set level inference



Sensitivity

Regional specificity

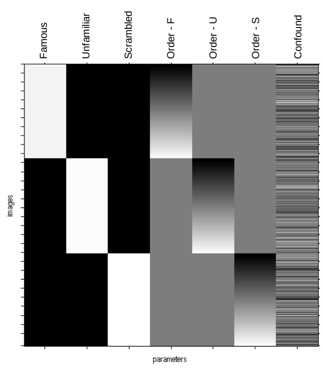


Peak level test: height of local maxima

Cluster level test: spatial extent above u

Set level test: number of clusters above u

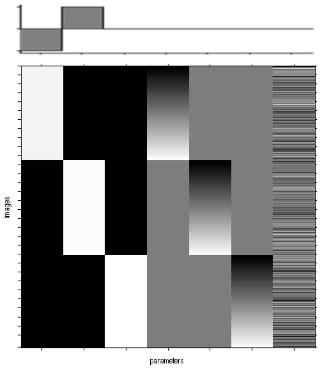
Design matrix for the practical demo



Famous < Unfamiliar

UCL

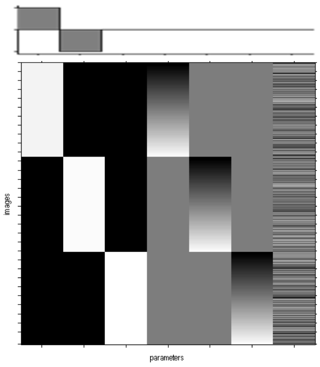
$$c^T = [-1 \quad 1]$$



Famous > Unfamiliar

UCL

$$c^T = [1 \quad -1]$$



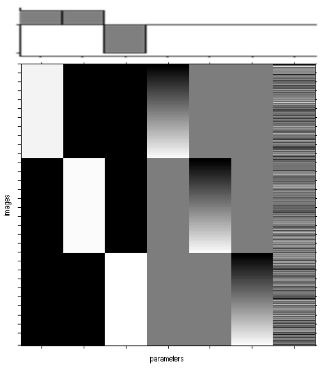
Faces > Scrambled

UCL

$$\frac{1}{2}\left[\hat{\beta}_1 + \hat{\beta}_2\right] > \hat{\beta}_3$$

$$0.5\hat{\beta}_1 + 0.5\hat{\beta}_2 - \hat{\beta}_3 > 0$$

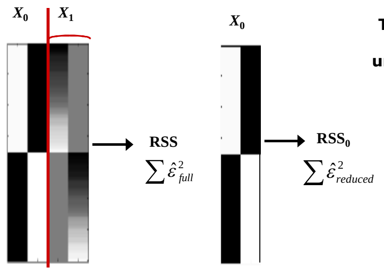
$$c^T = [0.5 \quad 0.5 \quad -1]$$



Extra-sum-of-squares and *F*-test

Model comparison: *Full vs. Reduced model?*

Null Hypothesis H_0: True model is X_0 (reduced model)



$$\sum \hat{\varepsilon}^2_{full} \qquad \sum \hat{\varepsilon}^2_{reduced}$$

Test statistic: ratio of explained and unexplained variability (error)

$$F \propto \frac{RSS_0 - RSS}{RSS}$$

$$F \propto \frac{ESS}{RSS} \sim F_{\nu_1,\nu_2}$$

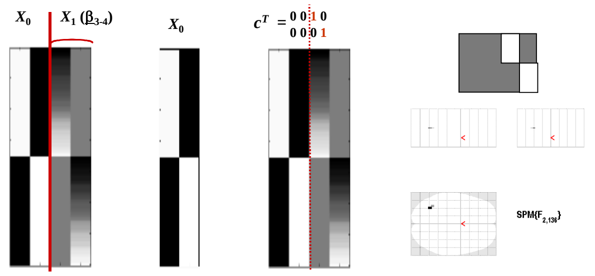
$\nu_1 = \mathrm{rank}(X) - \mathrm{rank}(X_0)$
$\nu_2 = N - \mathrm{rank}(X)$

F-test and multidimensional contrasts

Tests multiple linear hypotheses:

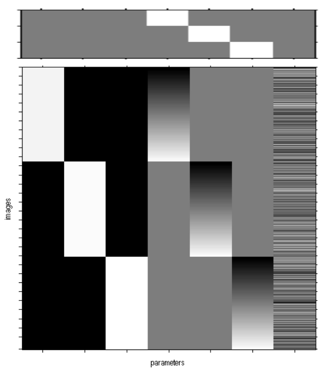
H_0: True model is X_0 | H_0: $\beta_3 = \beta_4 = 0$ | **test H_0: $c^T\beta = 0$?**

$$c^T = \begin{matrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

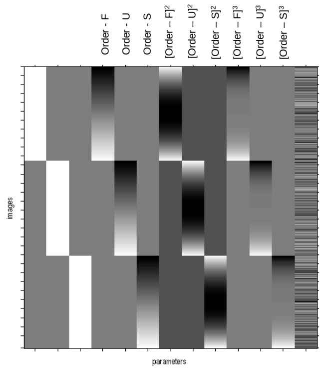


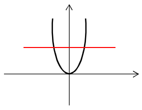
Full or reduced model?

Is there any effect of order ?



GLM and non-linear effects





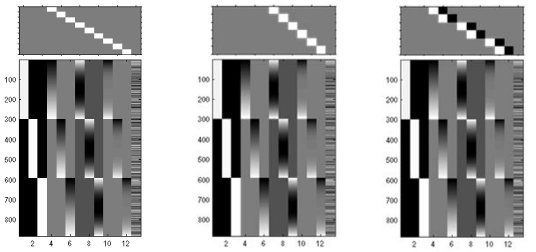
Although GLM is a linear tool it also allows to model non-linear effects of experimental variables by including squared, cubed etc. versions of the corresponding regressor (cf. Taylor expansion).

Many different questions can be tested

Is there any effect of order linear or nonlinear ?

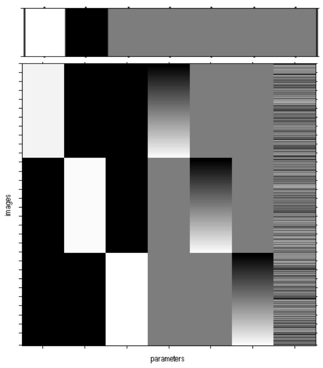
Is there nonlinear effect of order ?

Are there any differences in order effect between conditions?



F-test as a two-tailed t-test

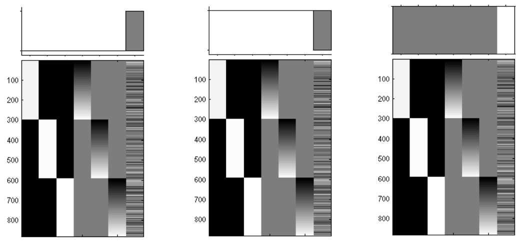
Famous<>Unfamiliar



Positive correlation with the confound (T)

Negative correlation with the confound (T)

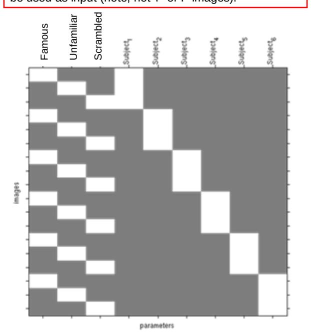
Any effect of confound (F)



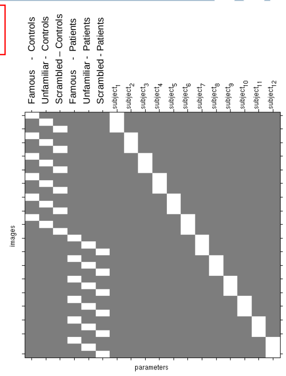
Note that p<0.05 for F-test corresponds to p<0.1 for T-test

Group-level analysis – exactly the same idea

Images exported directly from within-subject average or beta/contrast images from within-subject GLM can be used as input (note, not T- or F-images).



One-way ANOVA within-subject



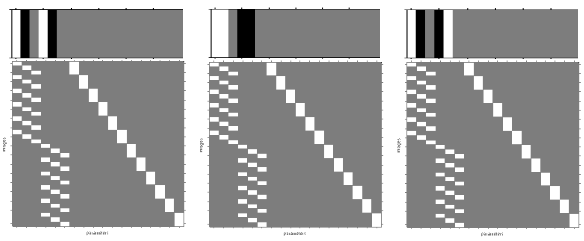
Flexible Factorial

Factorial design at the group level

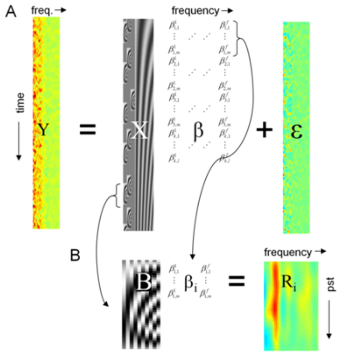
Main effect of familiarity

Main effect of disease

Interaction



Convolution modelling for M/EEG power



Litvak et al. Neuroimage 2013

Take home messages

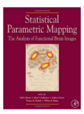
- The GLM is a flexible framework allowing to model multiple experimental factors simultaneously and disentangle their effects.
- The parametric framework allows for both peak-level and cluster-level inference. Thus, using cluster-based statistics possibly more sensitive to physiological effects in sensor-level or time-frequency data does not necessarily require randomisation.
- It is possible to easily incorporate nonlinear effects in a GLM and many other extensions are also possible.
- **Non-parametric – more flexibility with test statistic choice, limited design choice.**
- **Parametric – more flexibility with design choice, limited test statistic choice.**

The important thing is to use a valid method, whether parametric or non-parametric.

Thanks to

- Guillaume Flandin
- Stefan Kiebel
- Rik Henson

Thank you for your attention



Suggested further reading:

- http://www.fil.ion.ucl.ac.uk/spm/doc/biblio/Keyword/GLM.html
- “Statistical Parametric Mapping” book, Academic Press, 2006
- R. Henson, W. Penny. ANOVAs and SPM. http://www.mrc-cbu.cam.ac.uk//personal/rik.henson/personal/HensonPenny_ANOVA_03.pdf