

# Forward and inverse modelling of EEG and MEG

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# Overview

Motivation and background

Forward modeling

- Source model

- Volume conductor model

Inverse modeling - general

- Single and multiple dipole fitting

- Distributed source models

- Beamforming methods

Inverse modeling - independent components

Summary

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## **Motivation and background**

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### Inverse modeling - independent components

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# Motivation 1

## Strong points of EEG and MEG

- Temporal resolution ( $\sim 1$  ms)

- Characterize individual components of ERP

- Oscillatory activity

- Disentangle dynamics of cortical networks

## Weak points of EEG and MEG

- Measurement on outside of brain

- Overlap of components

- Low spatial resolution

## Motivation 2

If you find a ERP/ERF component, you want to characterize it in physiological terms

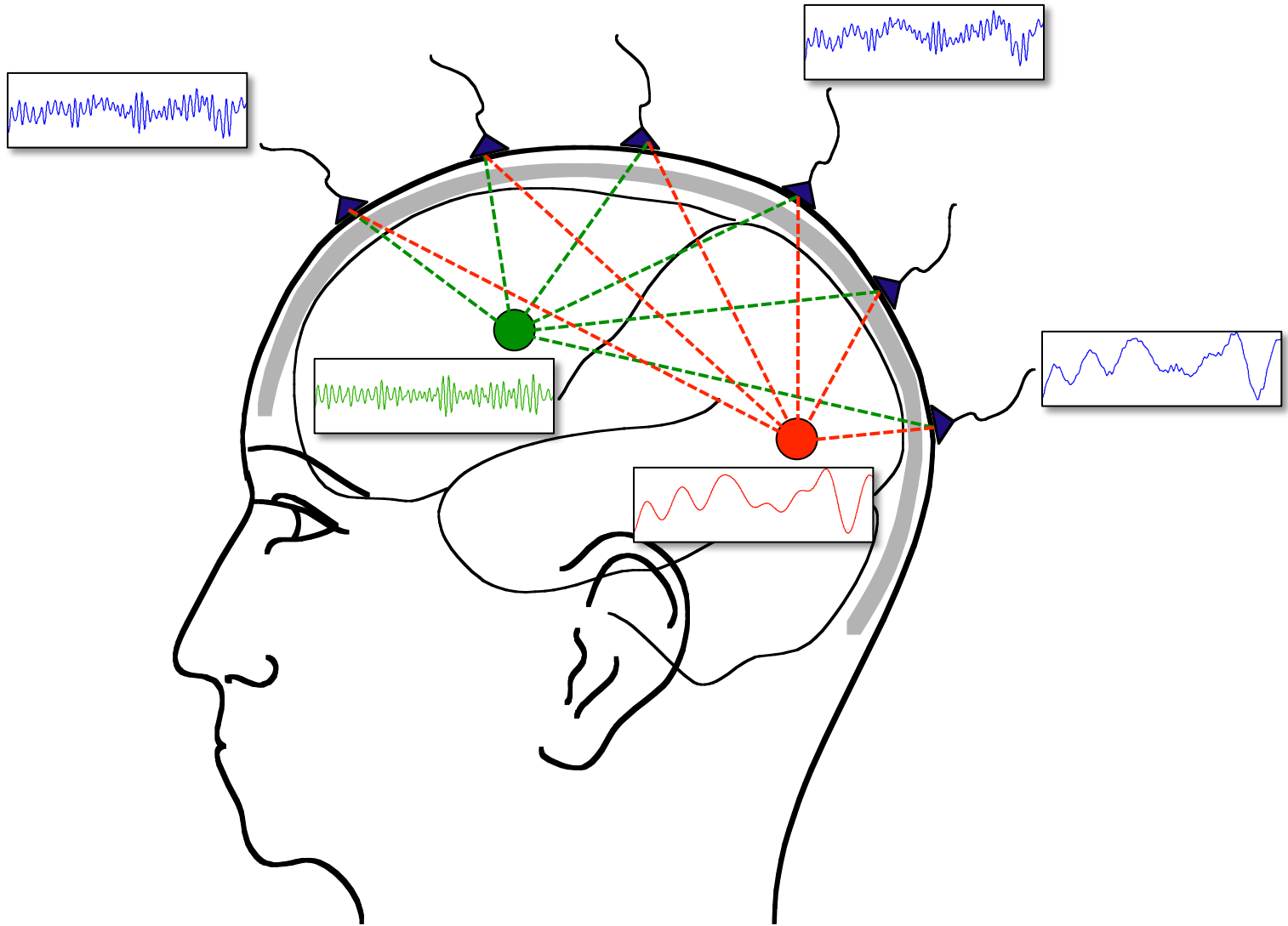
Time or frequency are the “natural” characteristics

“Location” requires interpretation of the scalp topography

Forward and inverse modeling helps to interpret the topography

Forward and inverse modeling helps to disentangle overlapping source timeseries

# Superposition of source activity



# Superposition of source activity

Varying “visibility” of each source to each channel

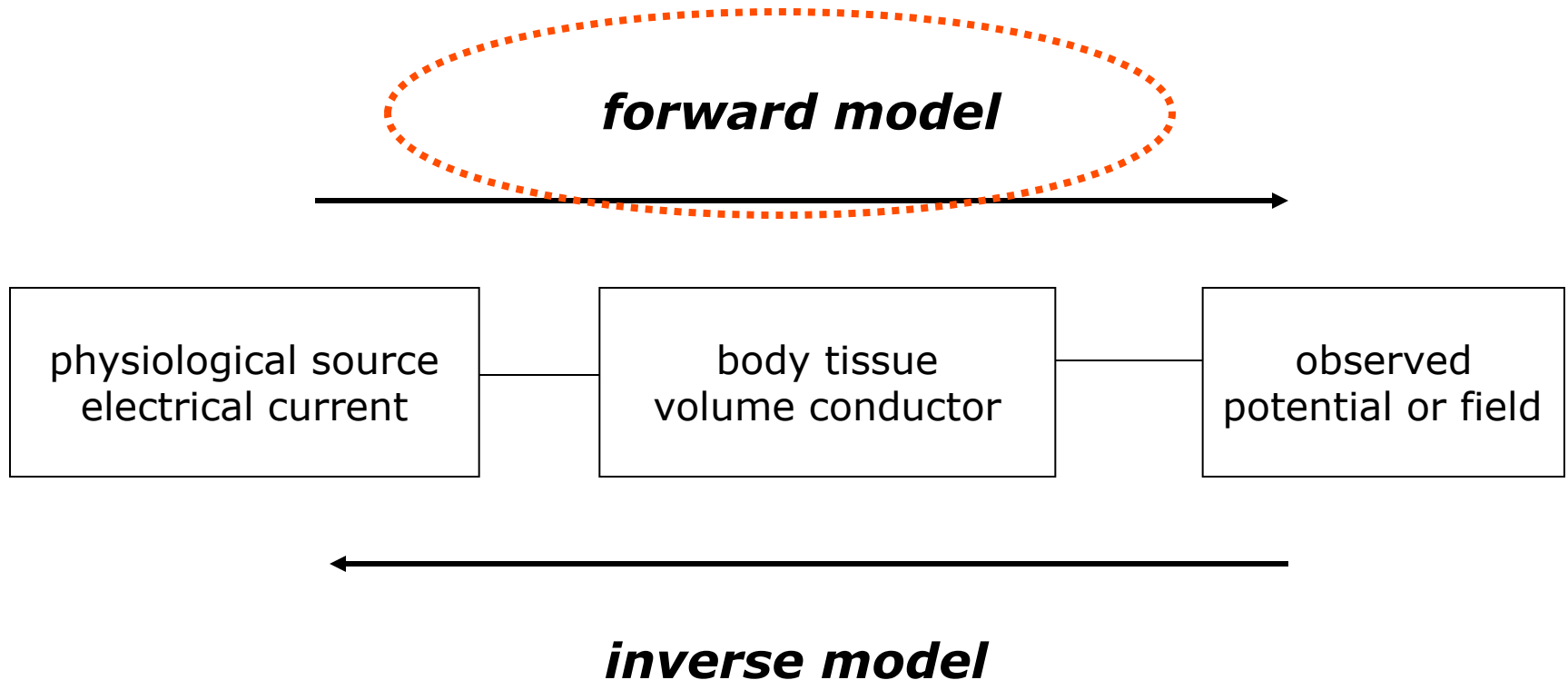
Timecourse of each source contributes to each channel

The contribution of each source depends on its “visibility”

Activity on each channel is a superposition of all source activity



# Biophysical source modelling: overview



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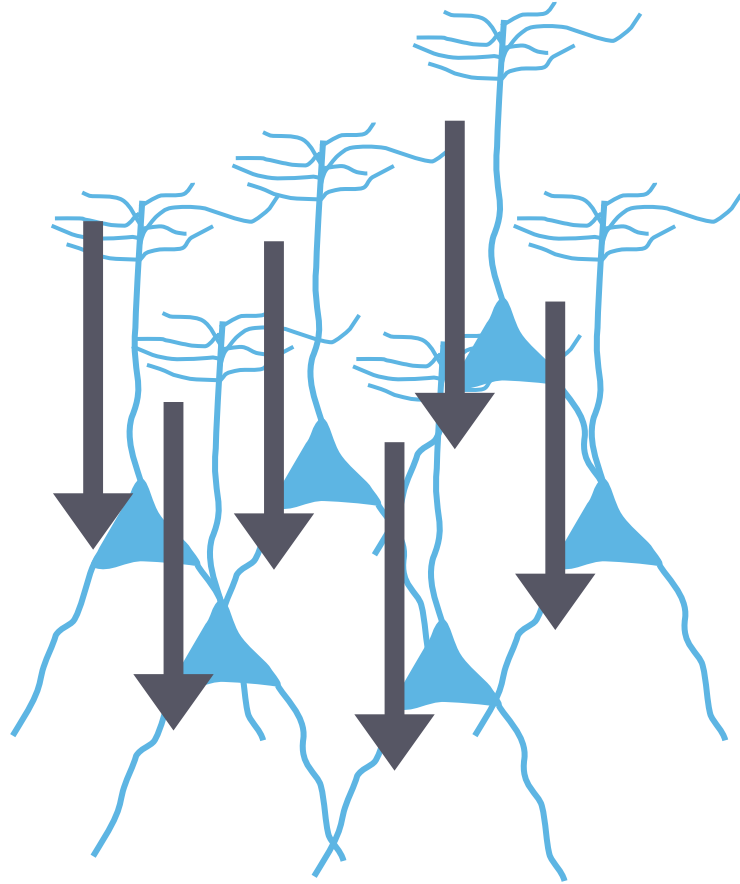
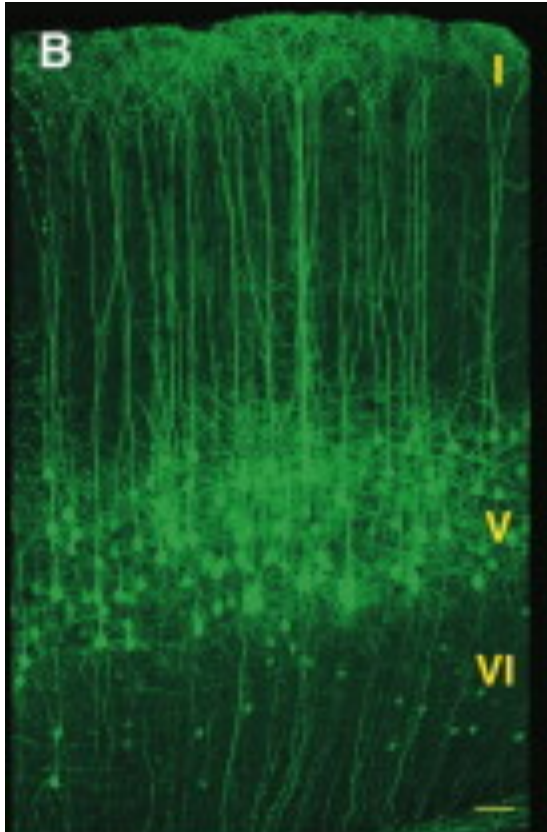
Distributed source models

Beamforming methods

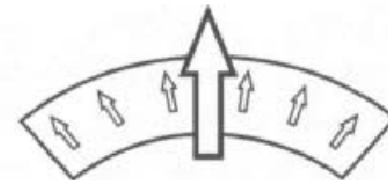
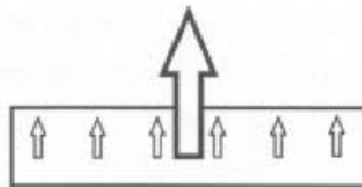
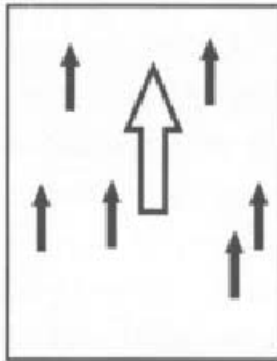
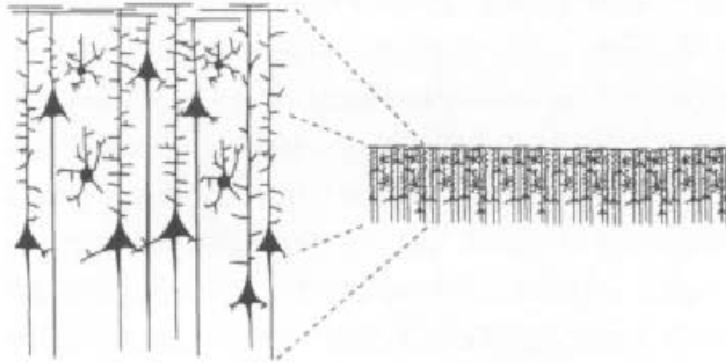
Inverse modeling - independent components

Summary

What produces the electric current



# Equivalent current dipoles



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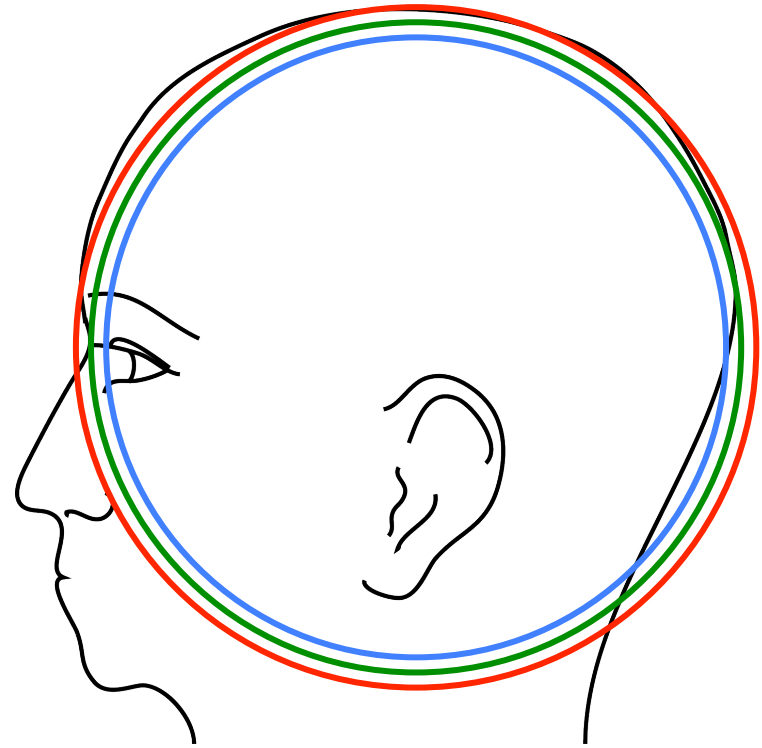
# Volume conductor

described electrical properties of tissue

describes geometrical model of the head

describes **how** the currents flow, not where they originate from

same volume conductor for EEG as for MEG, but also tDCS, tACS, TMS, ...



# Volume conductor

Computational methods for volume conduction problem that allow for realistic geometries

BEM      *Boundary Element Method*

FEM      *Finite Element Method*

FDM      *Finite Difference Method*

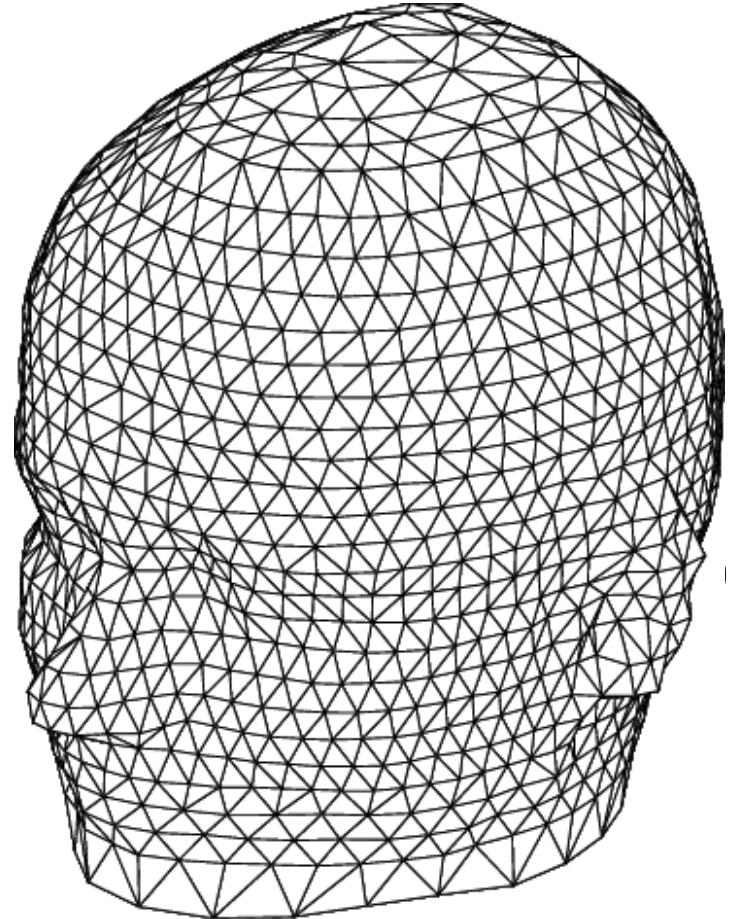
# Volume conductor: Boundary Element Method

Each compartment is  
homogenous  
isotropic

Important tissues

skin  
skull  
brain  
(CSF)

Triangulated surfaces  
describe boundaries





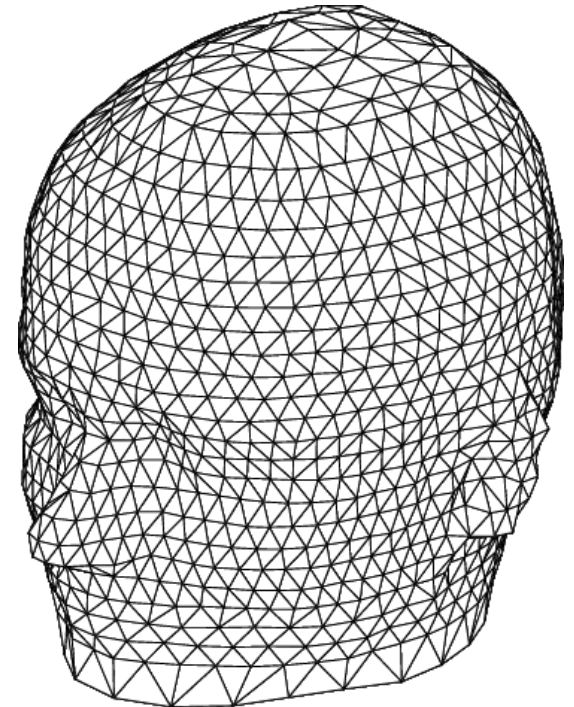
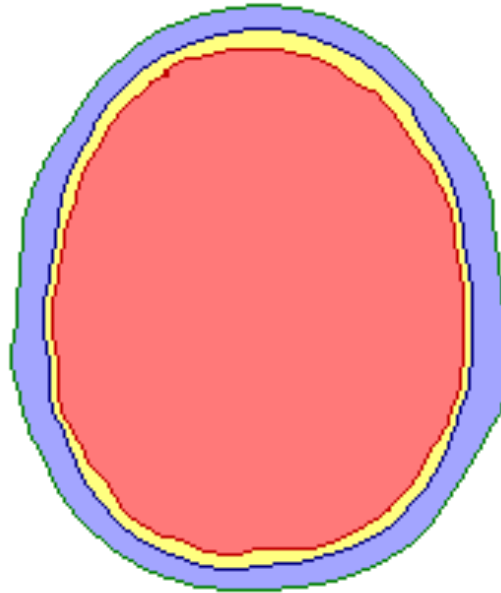
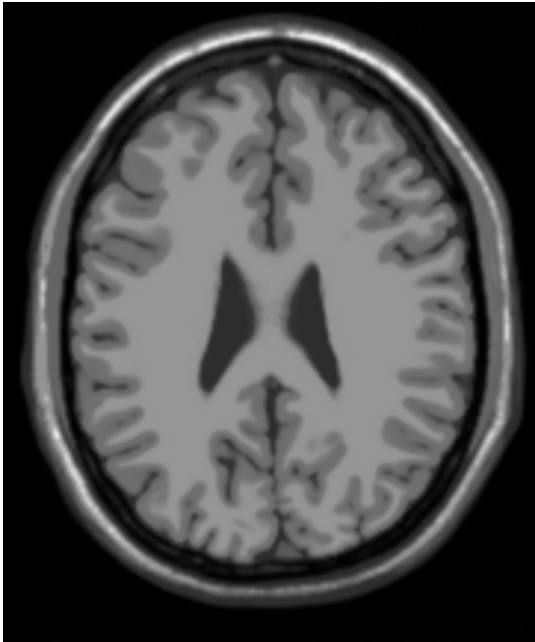
# Volume conductor: Boundary Element Method

## Construction of geometry

segmentation in different tissue types

extract surface description

downsample to reasonable number of triangles



# Volume conductor: Boundary Element Method

## Construction of geometry

- segmentation in different tissue types

- extract surface description

- downsample to reasonable number of triangles

## Computation of model

- independent of source model

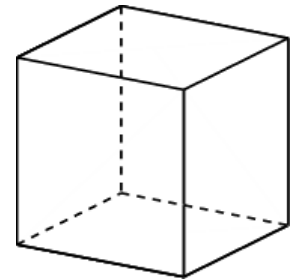
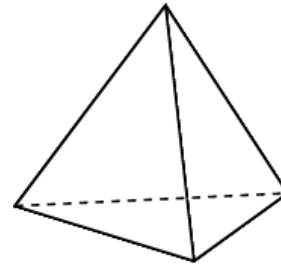
- only one lengthy computation

- fast during application to real data

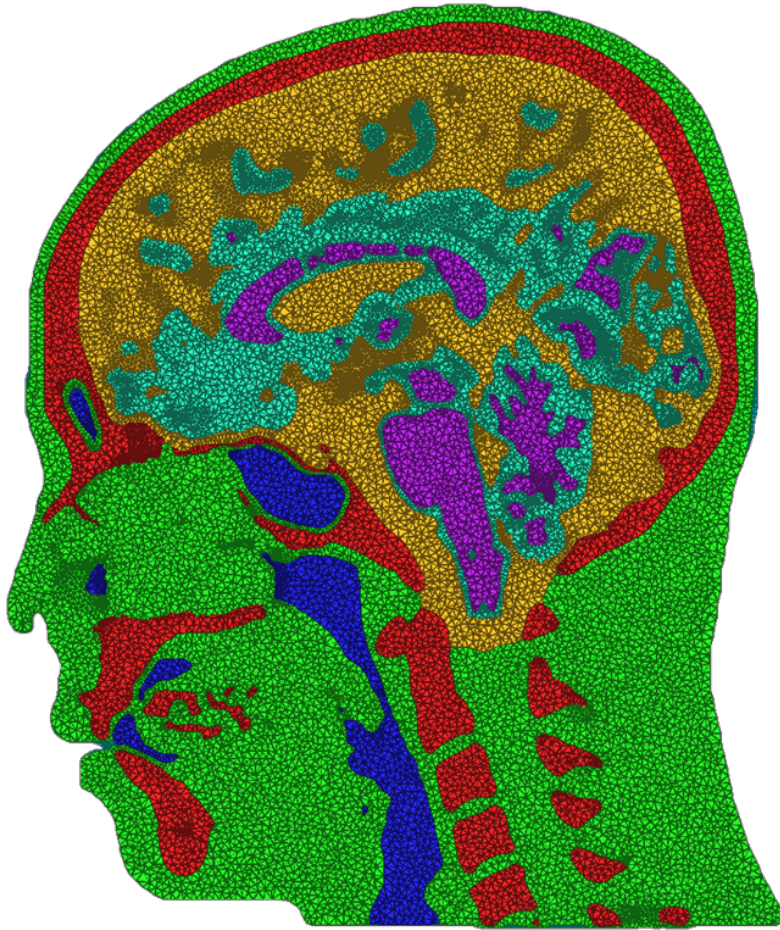
Can (almost) be arbitrary complex

# Volume conductor: Finite Element Method

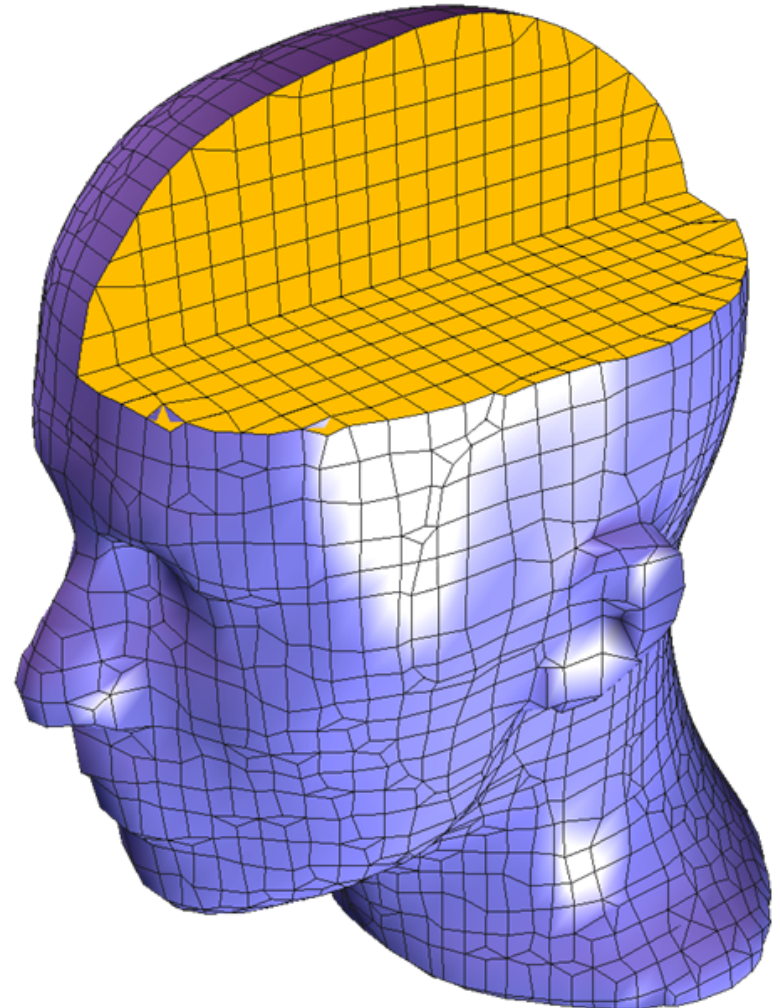
Tesselation of 3D volume in tetraeders or hexaheders



# Volume conductor: Finite Element Method



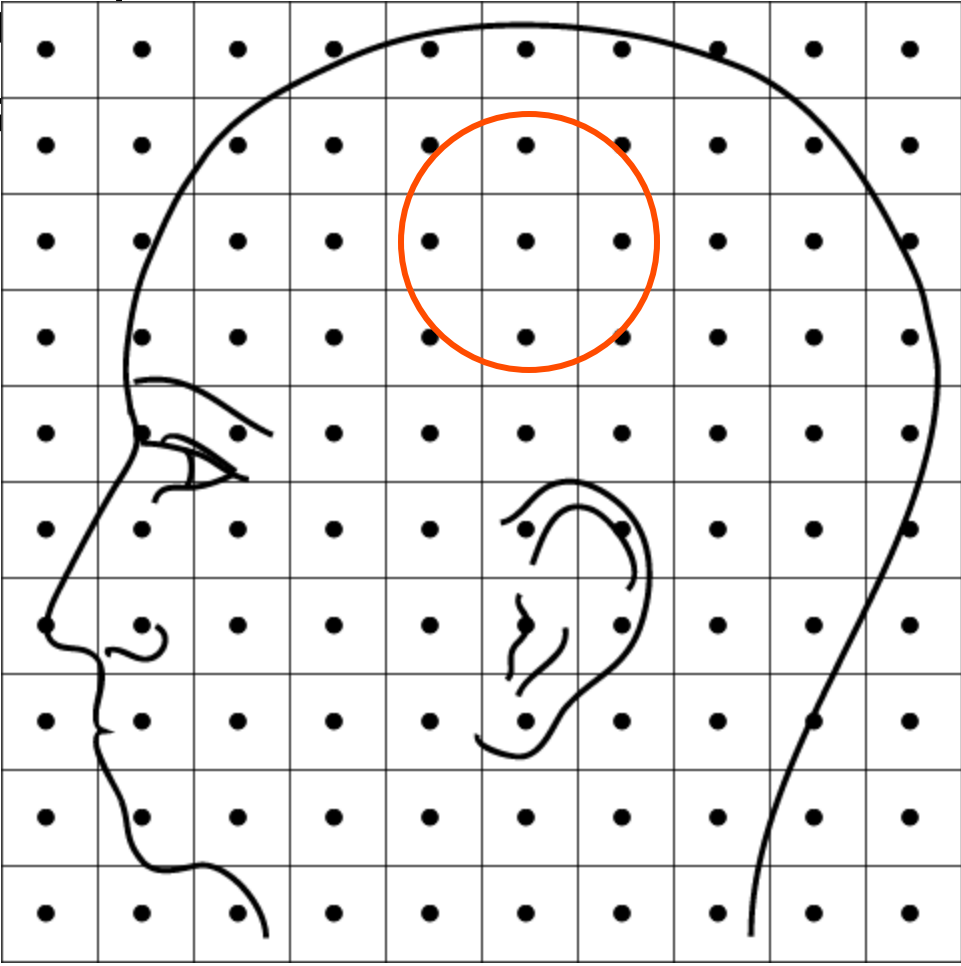
tetraeders



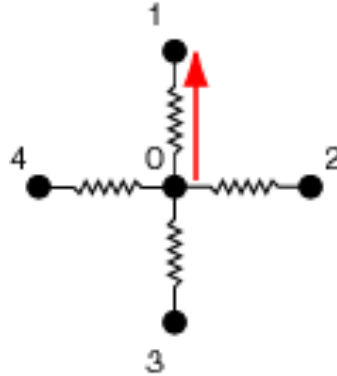
hexaheders

# Volume conductor: Finite Difference Method

Easy to con  
Not very us



# Volume conductor: Finite Difference Method



$$\left. \begin{aligned} I_1 + I_2 + I_3 + I_4 &= 0 \\ V &= I * R \end{aligned} \right\} \Rightarrow$$

$$\Delta V_1 / R_1 + \Delta V_2 / R_2 + \Delta V_3 / R_3 + \Delta V_4 / R_4 = 0 \quad \Rightarrow$$

$$(V_1 - V_0) / R_1 + (V_2 - V_0) / R_2 + (V_3 - V_0) / R_3 + (V_4 - V_0) / R_4 = 0$$

# Volume conductor: Finite Difference Method

Unknown potential  $V_i$  at each node

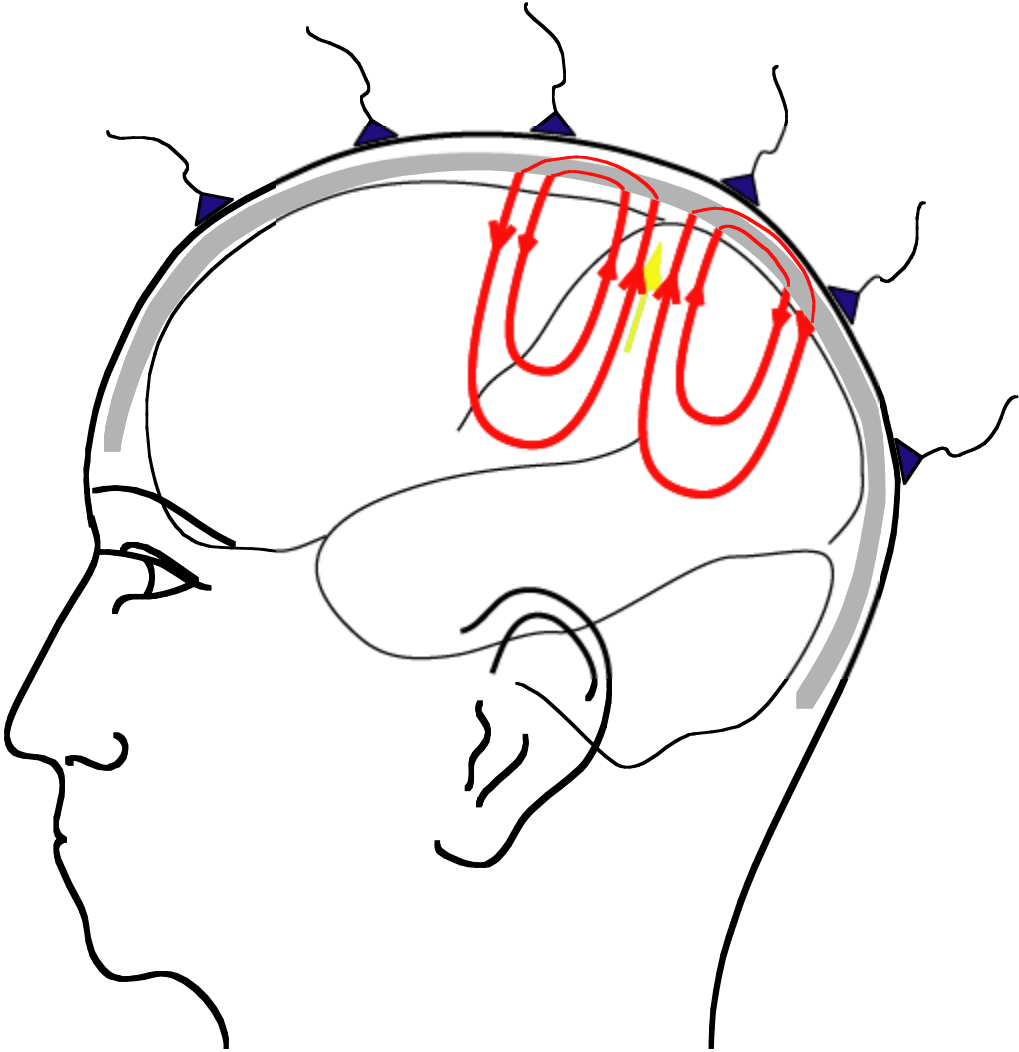
Linear equation for each node

approx.  $100 \times 100 \times 100 = 1.000.000$  linear equations  
just as many unknown potentials

Inject some current  $+I$  and  $-I$  at two of the nodes

Solve for unknown potential

# EEG volume conduction





# EEG volume conduction

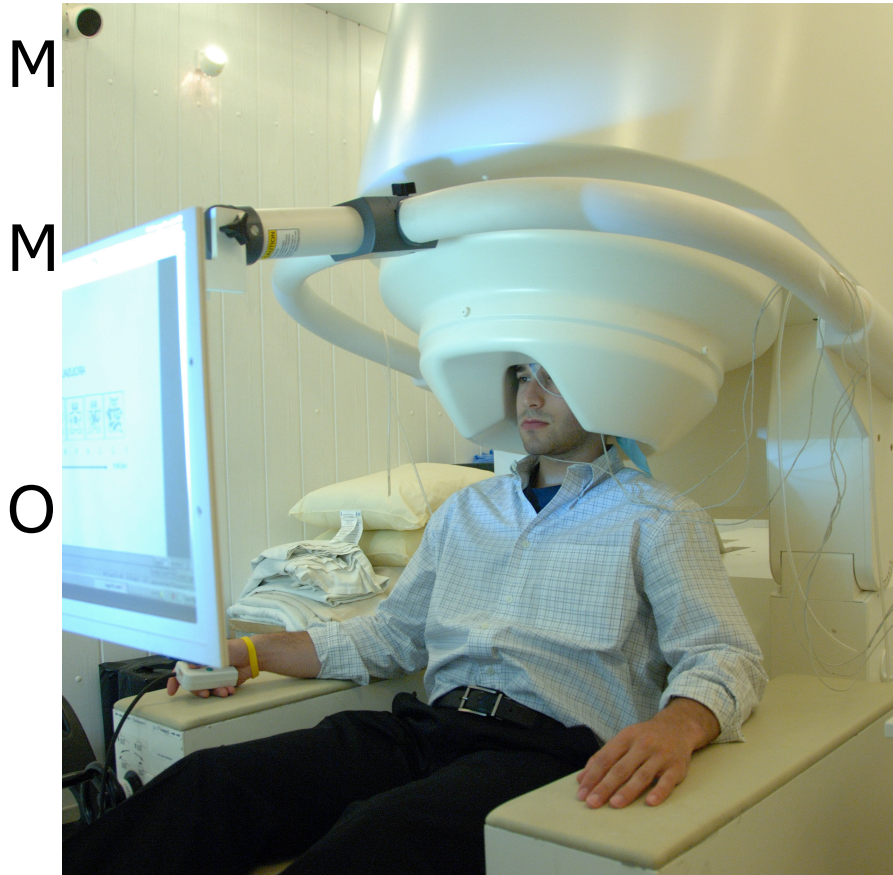
Potential difference between electrodes  
corresponds to current flowing through skin

Only tiny fraction of current passes through skull

Therefore the model should describe the skull and  
skin **as accurately as possible**

# MEG volume conduction

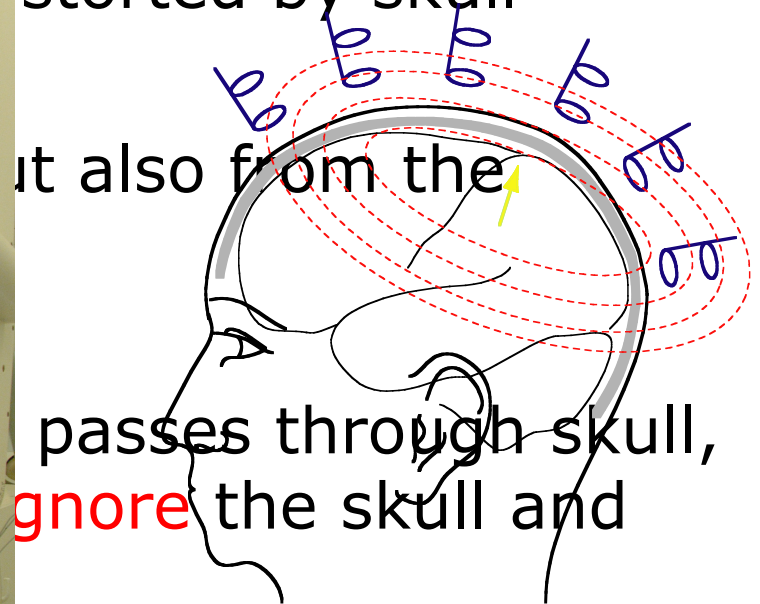
MEG measures magnetic field over the scalp



started by skull

it also from the

passes through skull,  
**gnore** the skull and



# Practical differences between EEG and MEG

fixed sensor positions in MEG

flexible cap in EEG

MEG requires head size to be known in analysis

using individual anatomical MRI

position of sensors is accurately known

EEG requires the electrode positions to be known  
in analysis

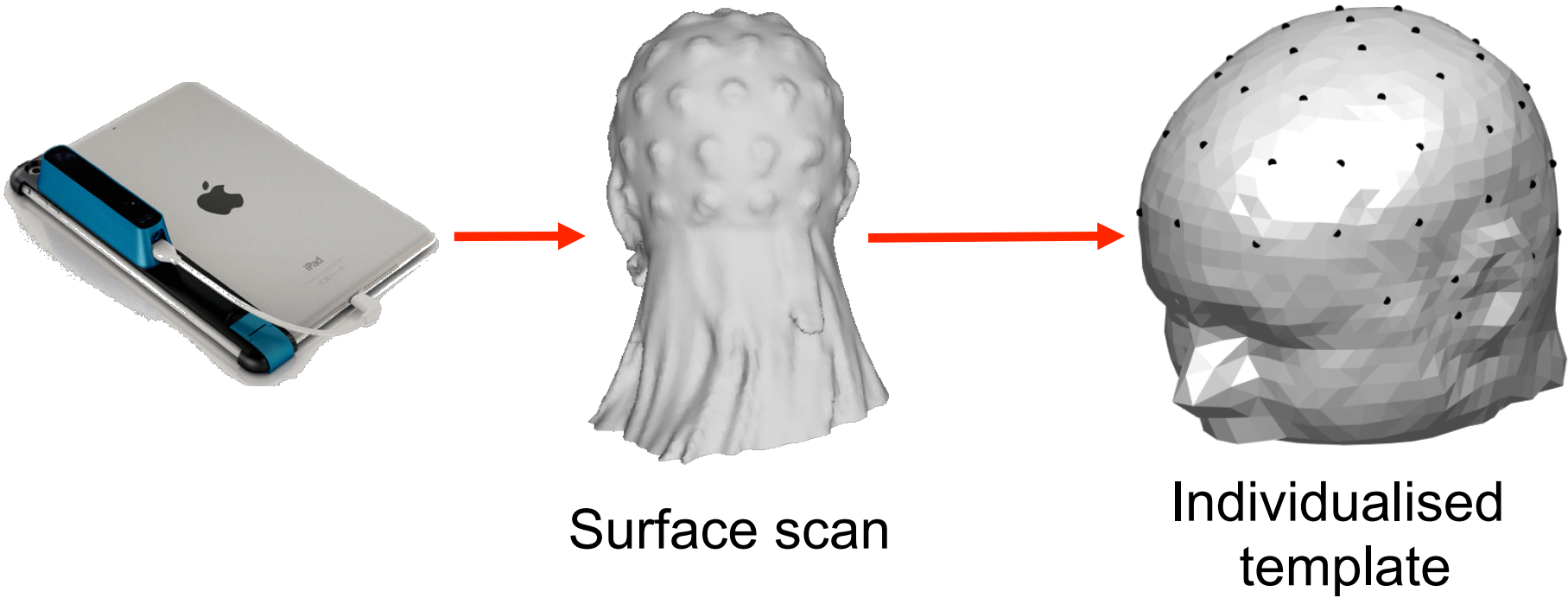
# Obtaining geometrical data



# 3D scanning instead of MRI



# 3D scanning - pipeline for EEG modelling



## Forward modeling – practical considerations

**Most accurate** source estimate using individual headmodels and electrode positions

**Decent accurate** source estimate with template headmodel and individual electrode positions

**Reasonably accurate** source estimate with template BEM headmodel and template electrodes

**Least accurate** source estimate with spherical model and template electrodes

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**Inverse modeling - general**

- Single and multiple dipole fitting

- Distributed source models

- Beamforming methods

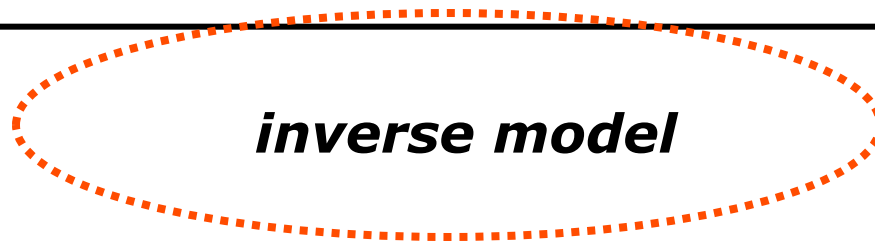
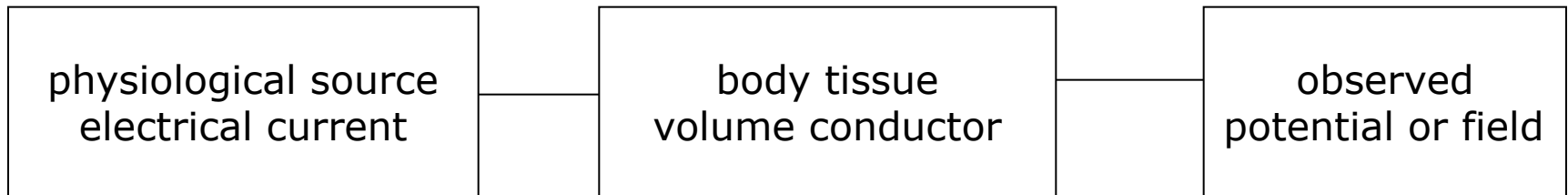
Inverse modeling - independent components

Summary



# Biophysical source modelling: overview

***forward model***



***inverse model***

# Inverse methods

## Single and multiple dipole models

Minimize error between model and measured potential/field

## Distributed source models

Perfect fit of model to the measured potential/field

Additional constraint on source smoothness, power or amplitude

## Spatial filtering

Scan the whole brain with a single dipole and compute the filter output at every location

Beamforming (e.g. LCMV, SAM, DICS)

Multiple Signal Classification (MUSIC)

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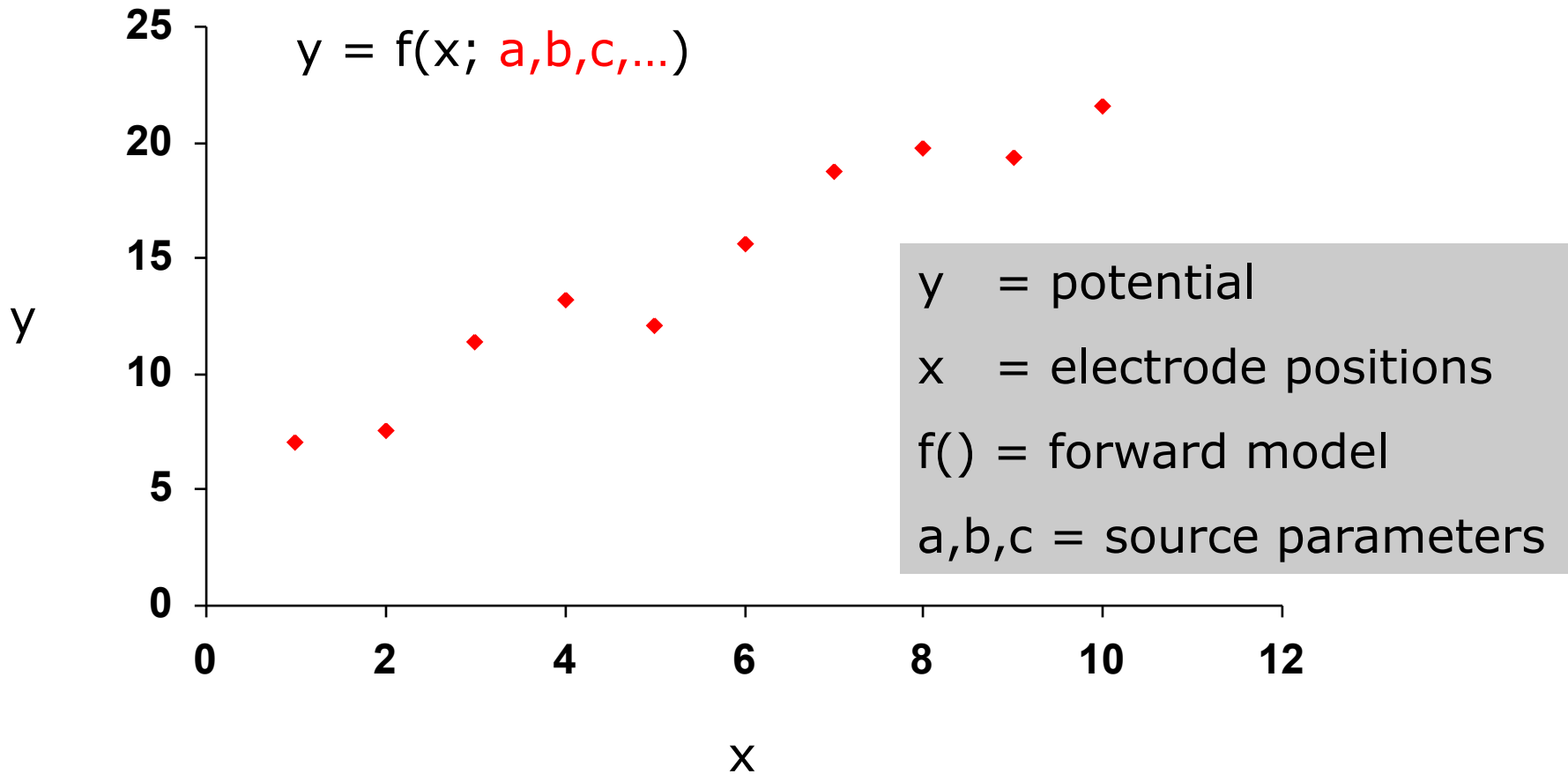
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# Single or multiple dipole models - Parameter estimation



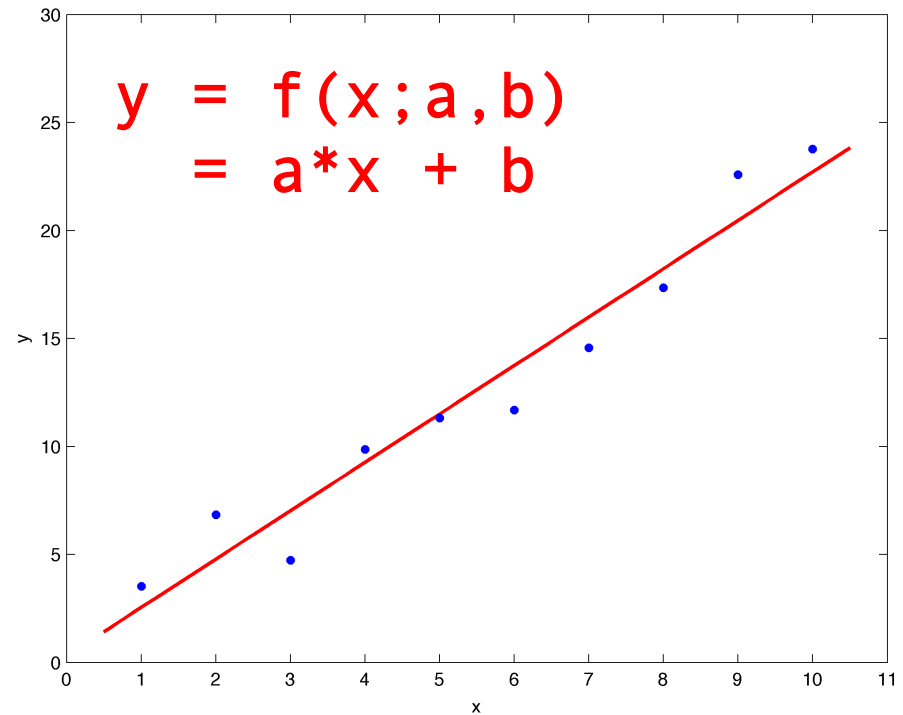
# Parameter estimation: dipole parameters

source model with  
few parameters

position  
orientation  
strength

compute the model  
data

minimize difference  
between actual and  
model data



# Non-linear parameters: grid search

One dimension, e.g. location along medial-lateral

100 possible locations

Two dimensions, e.g. med-lat + inf-sup

$100 \times 100 = 10.000$

Three dimensions

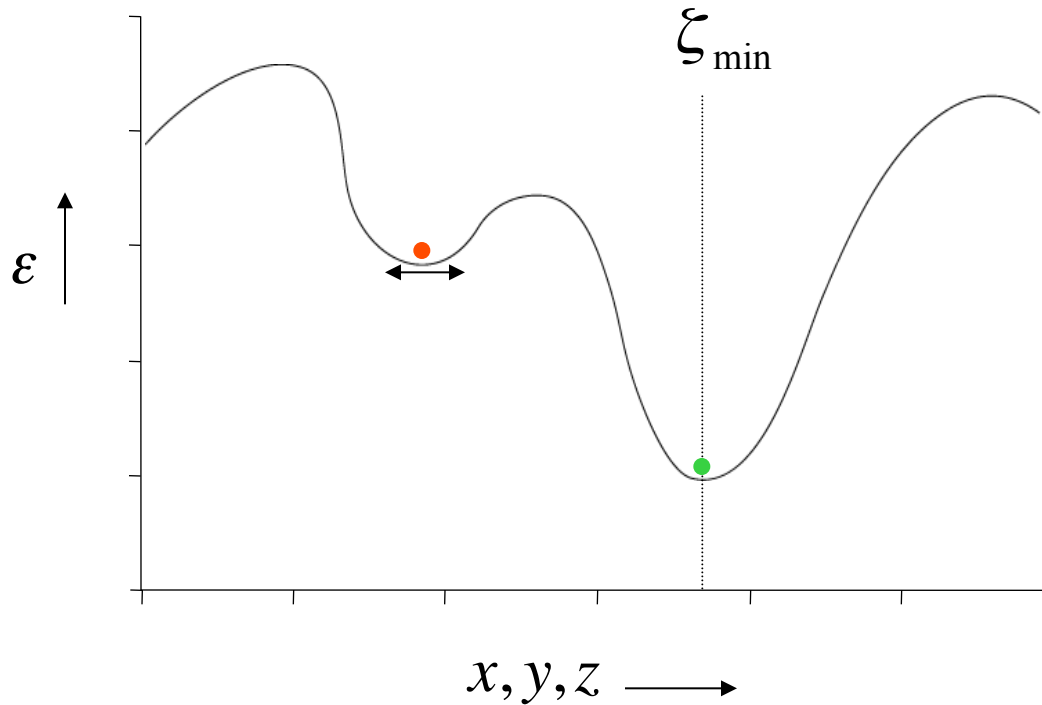
$100 \times 100 \times 100 = 1.000.000 = 10^6$

Two dipoles, each with three dimensions

$100 \times 100 \times 100 \times 100 \times 100 \times 100 = 10^{12}$

# Optimization of non-linear parameters

$$\text{error}(x, y, z) = \sum_{i=1}^N (Y_i(x, y, z) - V_i)^2 \Rightarrow \min_{x, y, z} (\text{error}(x, y, z))$$



# Single or multiple dipole models - Strategies

Single dipole:

scan the whole brain, followed by iterative optimization

Two dipoles:

scan with symmetric pair, use that as starting point for iterative optimization

More dipoles:

sequential dipole fitting



# Sequential dipole fitting for ERPs

Assume that activity starts “small”

explain earliest ERP component with single equivalent current dipole

Assume later activity to be more widespread

add ECDs to explain later ERP components

estimate position of new dipoles

re-estimate the activity of all dipoles

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# Distributed source model

Position of the source is **not estimated** as such

Pre-defined grid (3D volume or on cortical sheet)

Strength is estimated

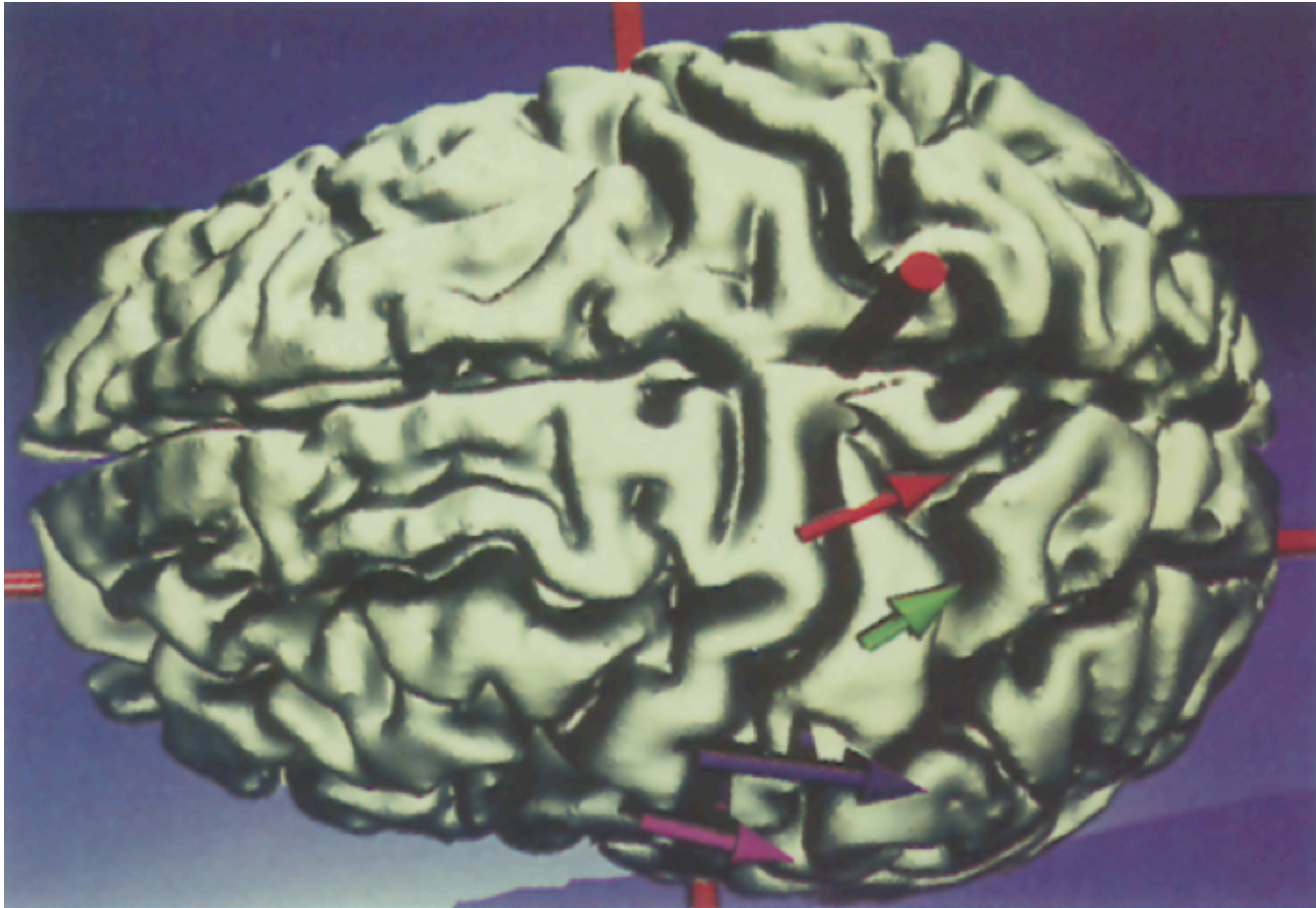
In principle easy to solve, however...

More “unknowns” (parameters) than  
“knowns” (measurements)

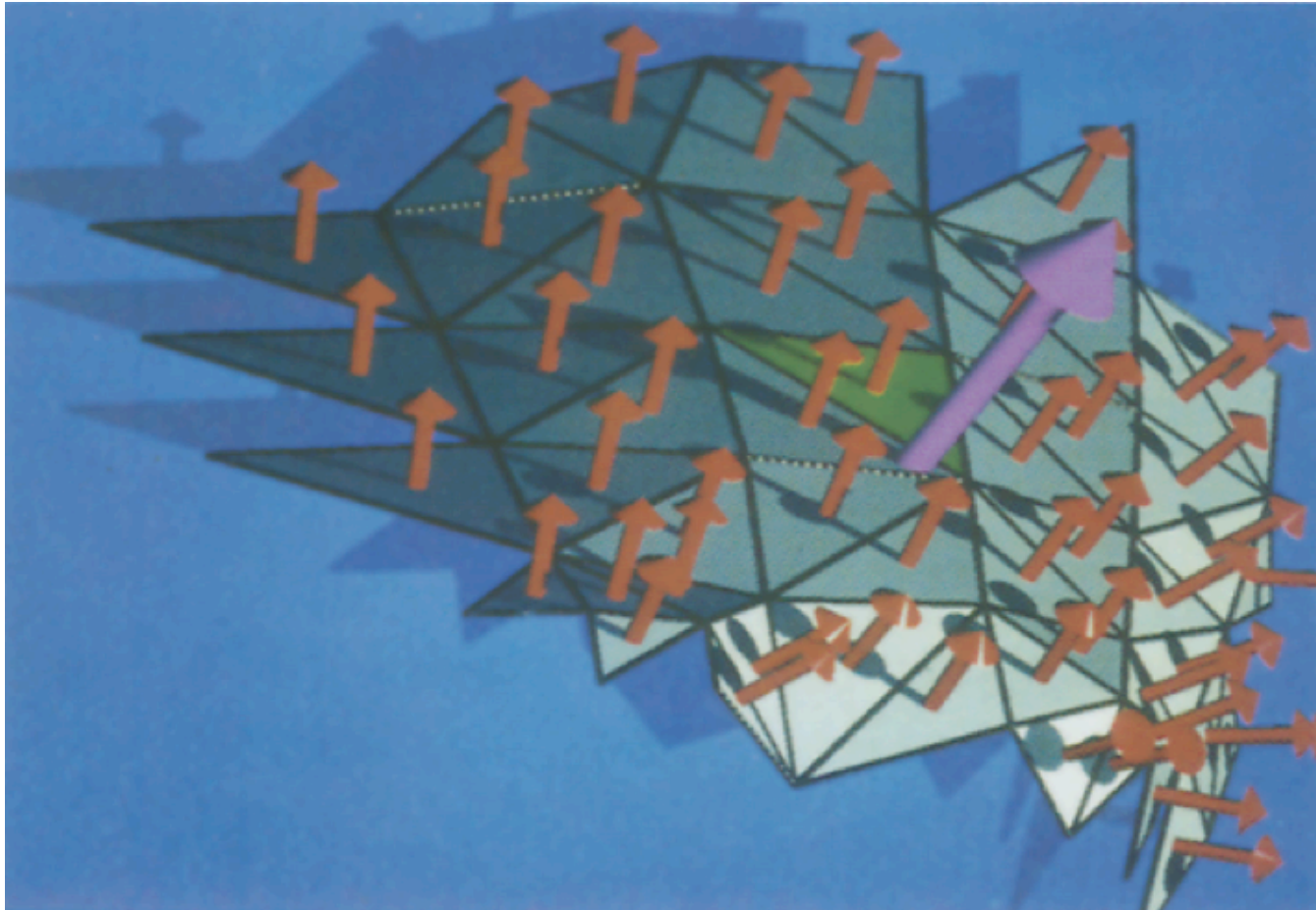
Infinite number of solutions can explain the data  
perfectly

Additional constraints required

# Distributed source model



# Distributed source model

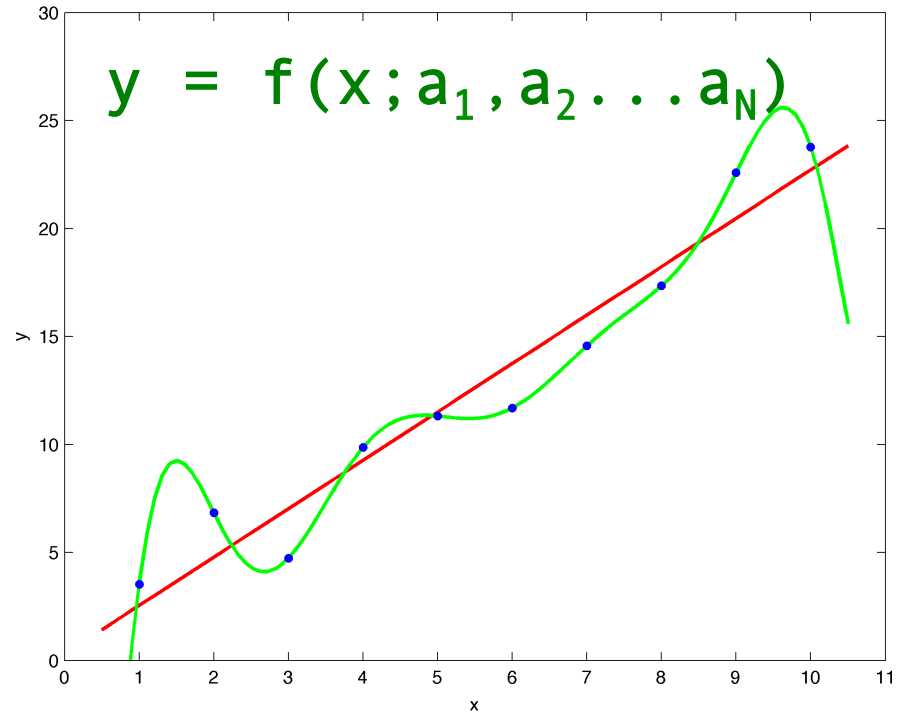


# Distributed source model: linear estimation

distributed source model  
with **many dipoles**  
throughout the whole  
brain

estimate the strength of  
all dipoles

data and noise can be  
perfectly explained



# Distributed source model: regularization


$$V = G \cdot q + \text{Noise}$$

$$\min_q \{ \|V - G \cdot q\|^2 \} = 0 \quad !!$$

Regularized linear estimation:

$$\rightarrow \min_q \{ \|V - G \cdot q\|^2 + \lambda \cdot \|D \cdot q\|^2 \}$$

  
mismatch with data

  
mismatch with prior  
assumptions

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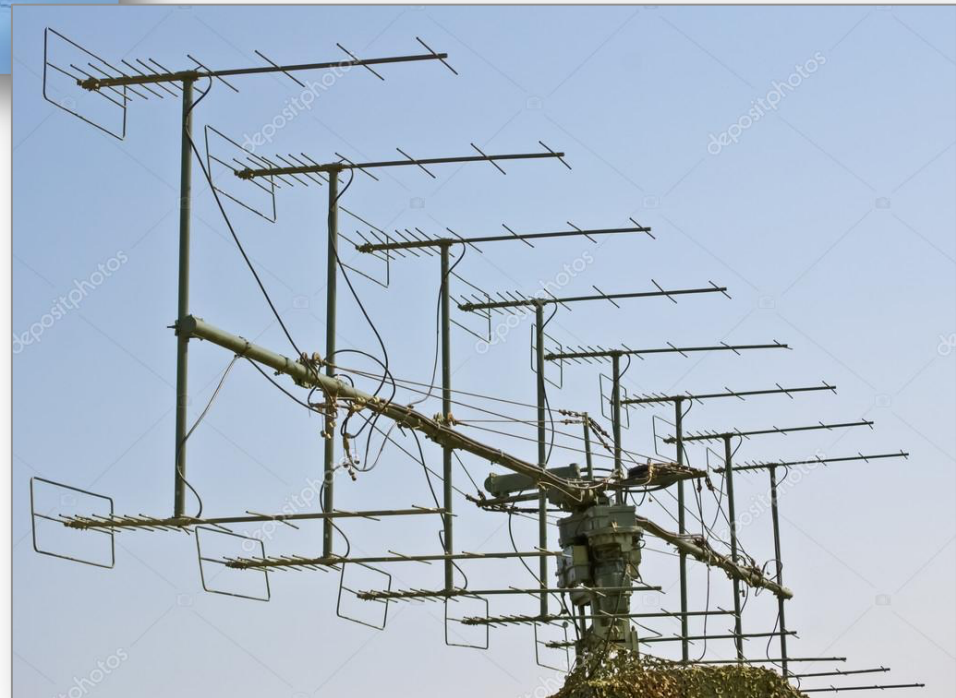
- Beamforming methods**

Inverse modeling - independent components

Summary



# Scanning with a beamformer filter



# Spatial filtering with beamforming

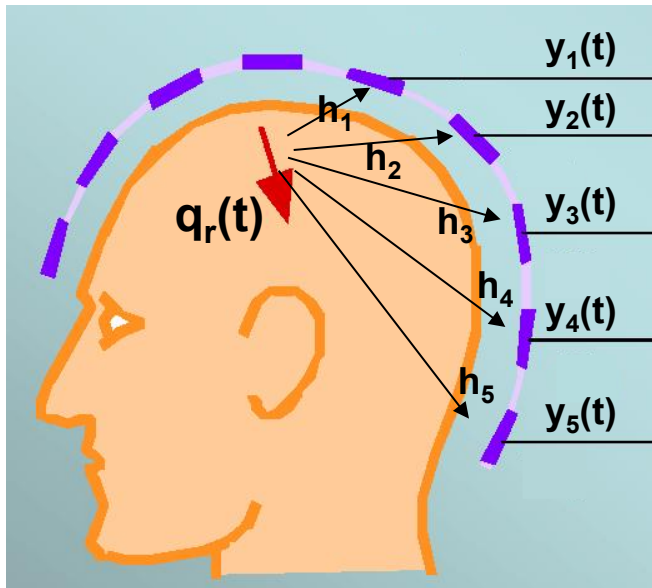
Position of the source is **not estimated** as such

Manipulate filter properties, not source properties

No explicit assumptions about source constraints  
(implicit: single dipole)

Assumption that sources that contribute to the data  
should be uncorrelated

# Beamformer ingredients: forward model



forward model

$$Y(t) = G * q(t)$$

The diagram shows a large red square labeled  $Y(t)$  on the left, followed by an equals sign, a vertical red bar labeled  $G$ , an asterisk, and a horizontal red bar labeled  $q(t)$  on the right. An arrow points from the text "forward model" to the  $G$  bar.

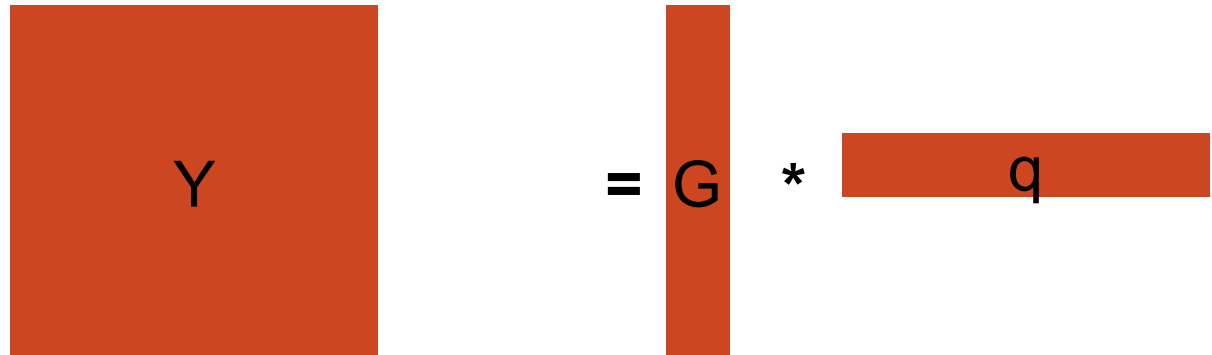
# Beamformer: the question revisited

What is the activity of a source  $\mathbf{q}$ , at a location  $\mathbf{r}$ , given the data  $\mathbf{Y}$ ?

*We know how to get from source to data:  $\mathbf{Y} = \mathbf{G} * \mathbf{q}$*

*We want to go from data to source:  $\mathbf{w}^T * \mathbf{Y} = \hat{\mathbf{q}}$*

$\mathbf{w}^T$  is called a spatial filter



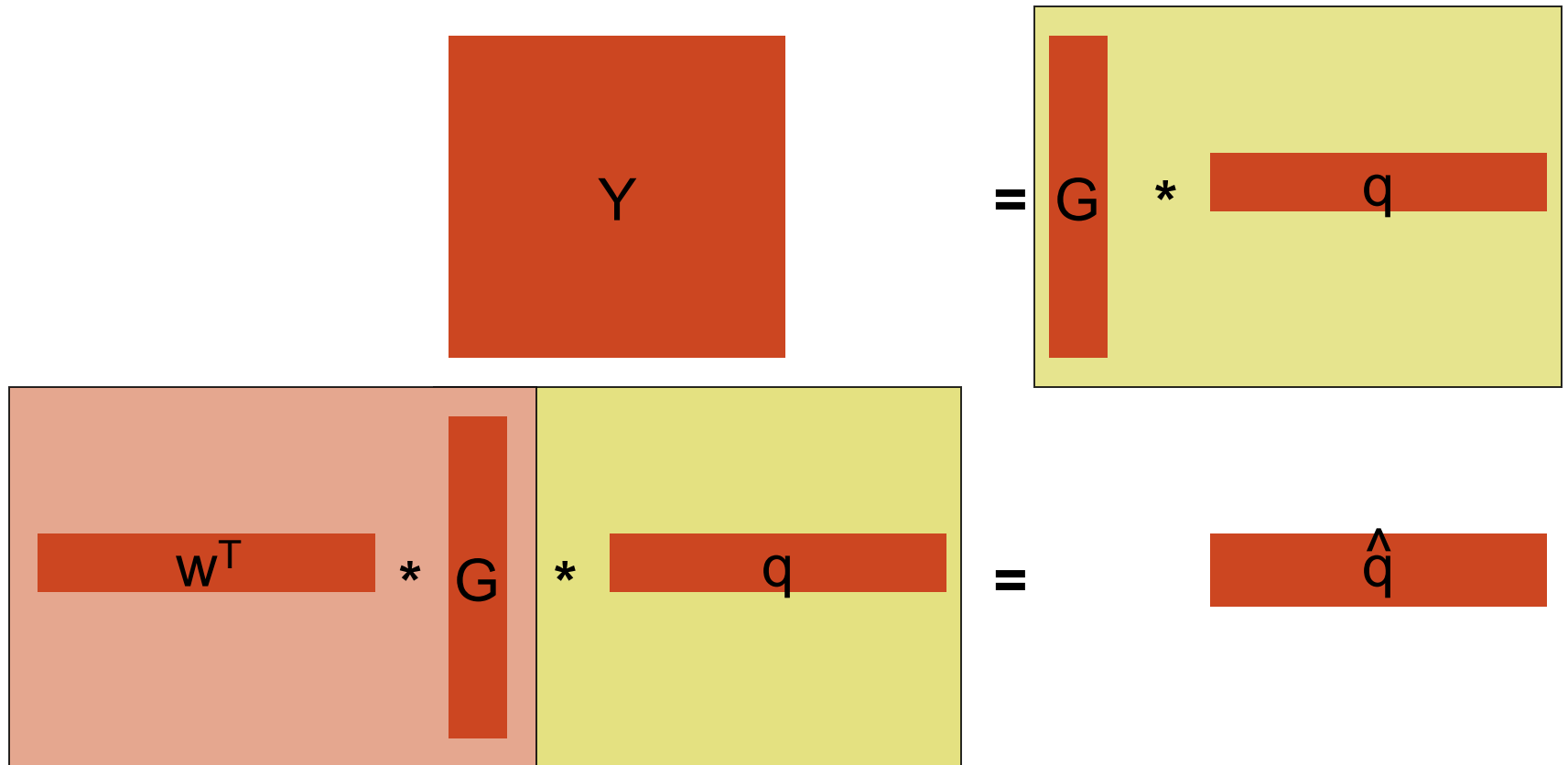
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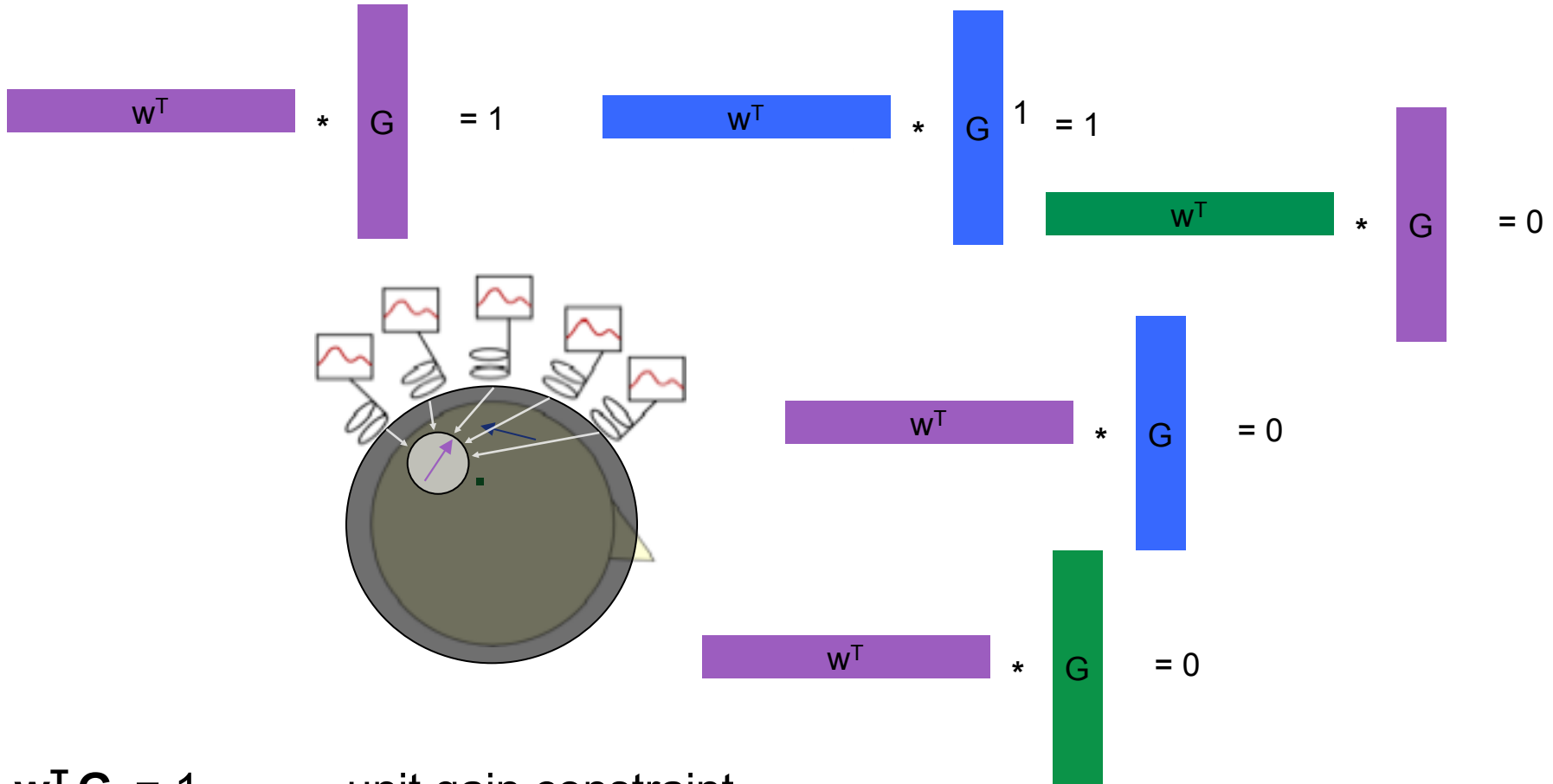
*We know how to get from source to data:  $\mathbf{Y} = \mathbf{G} * \mathbf{q}$*

*We want to go from data to source:  $\mathbf{w}^T * \mathbf{Y} = \hat{\mathbf{q}}$*

$\mathbf{w}^T$  is called a spatial filter



# What would we like a spatial filter to do?



$$w^T_i G_i = 1$$

$$w^T_i G_k = 0$$

unit gain constraint

cannot generally be fulfilled, hence we minimize the *variance* of the filter output

# Adaptive spatial filter: minimum variance constraint

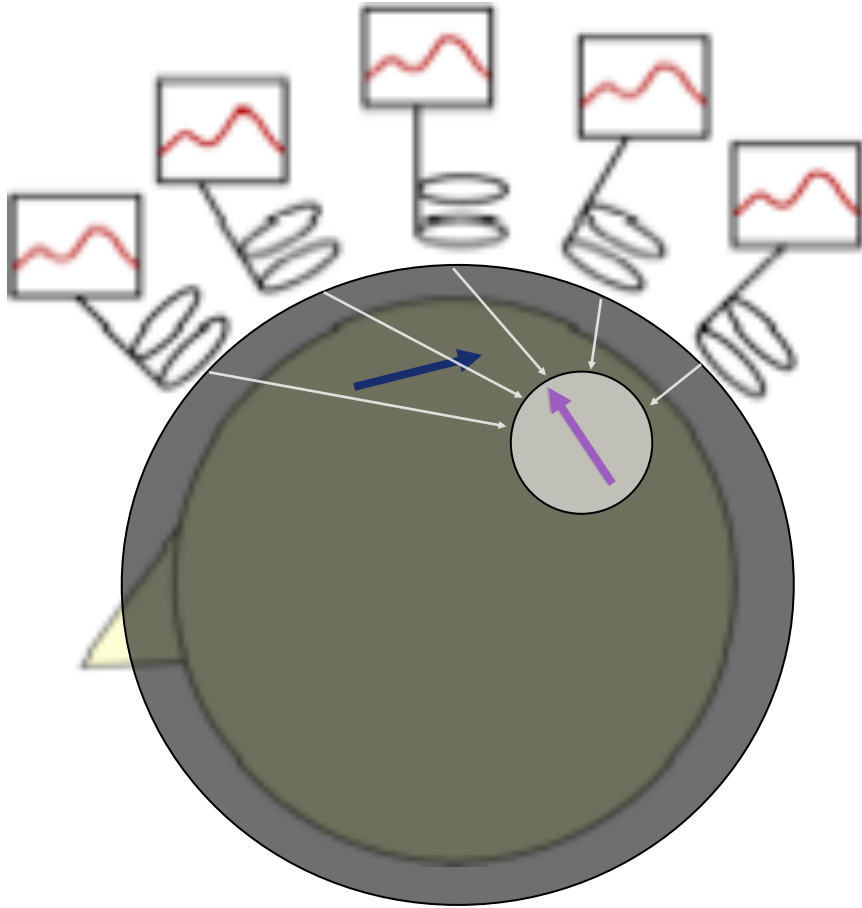
$$w^T * Y = q$$

$$\text{var}(q) = w^T * Y * C_y * Y^T * w$$

↓ minimized by:

$$w^T = [G^T C_y^{-1} G]^{-1} G^T C_y^{-1}$$

# Spatial filtering with beamforming



$$\mathbf{w}^T * \mathbf{G} = 1$$

$$\mathbf{w}^T * \mathbf{G} = 1$$

$$\mathbf{w}^T * \mathbf{G} = 0$$



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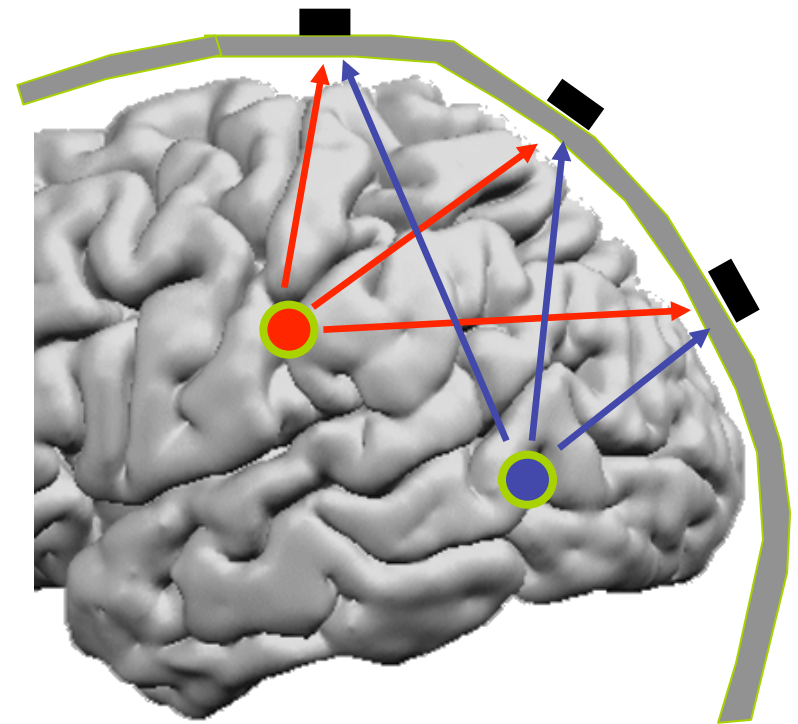
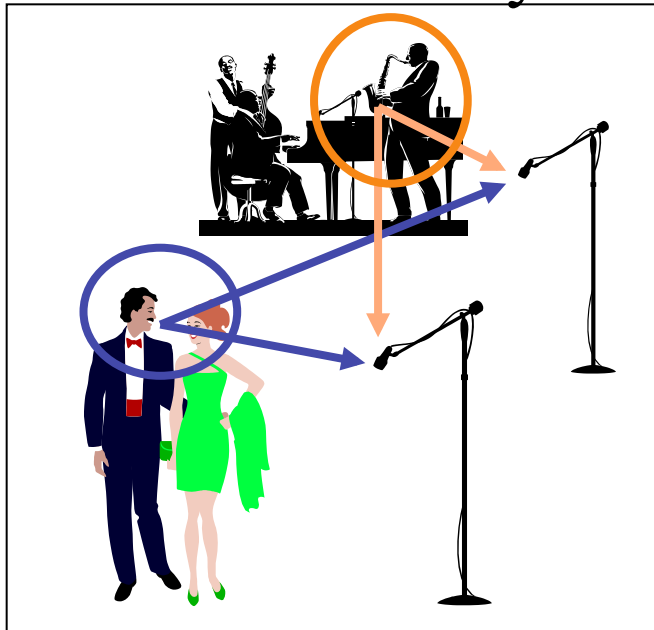
**Inverse modeling - independent components**

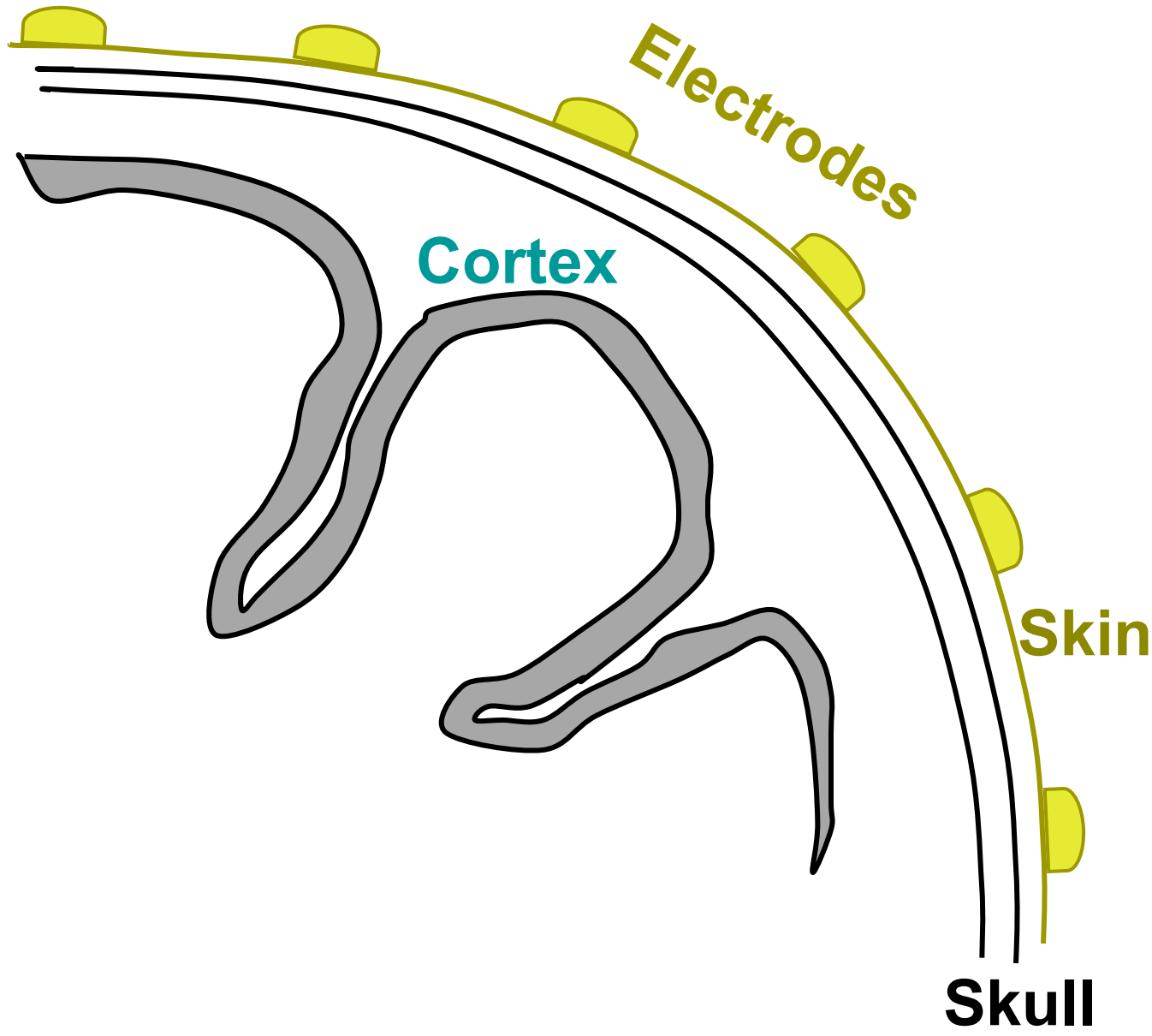
Summary

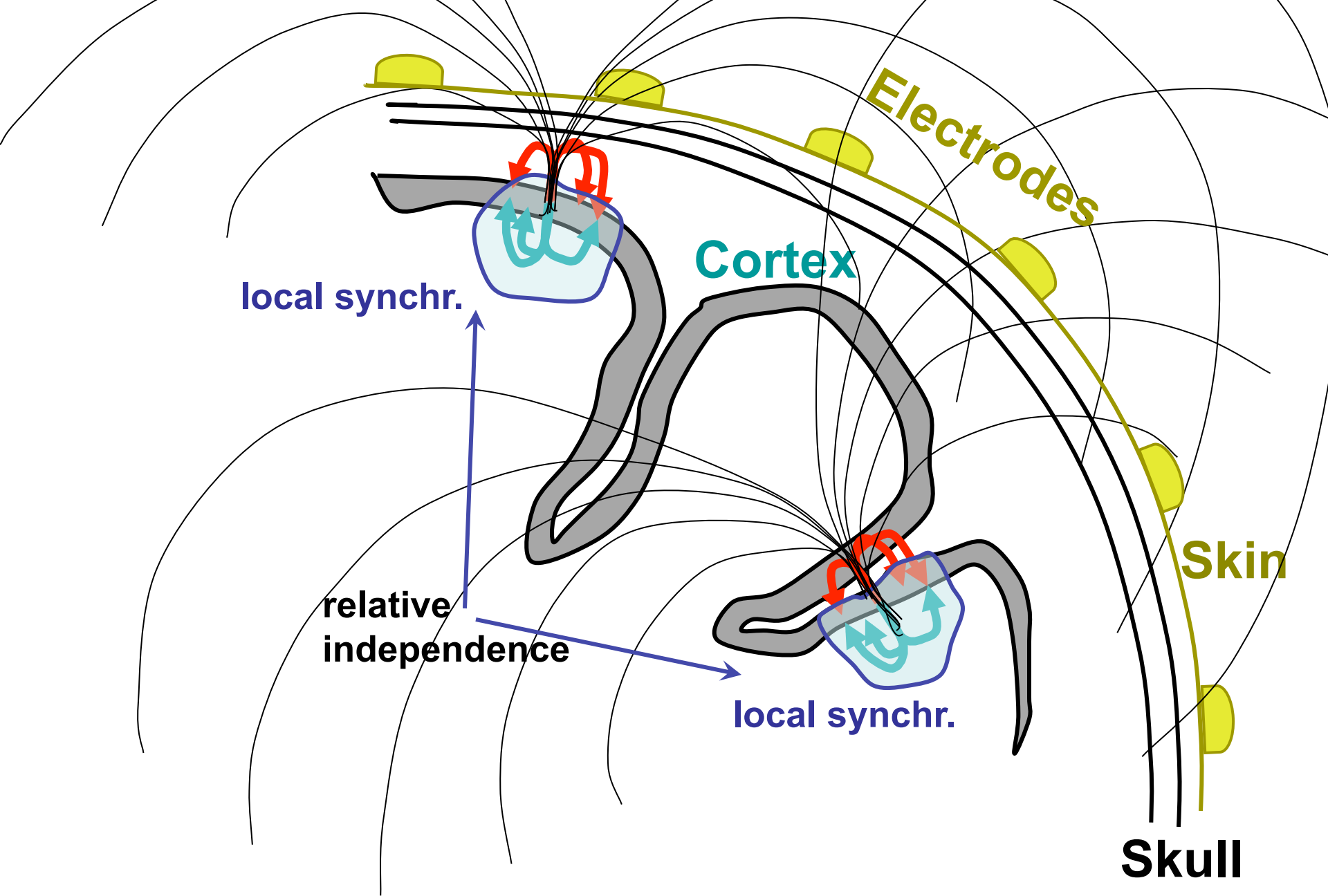
# Independent component analysis

Mixture of Brain source activity

## Cocktail Party







local synchr.

Cortex

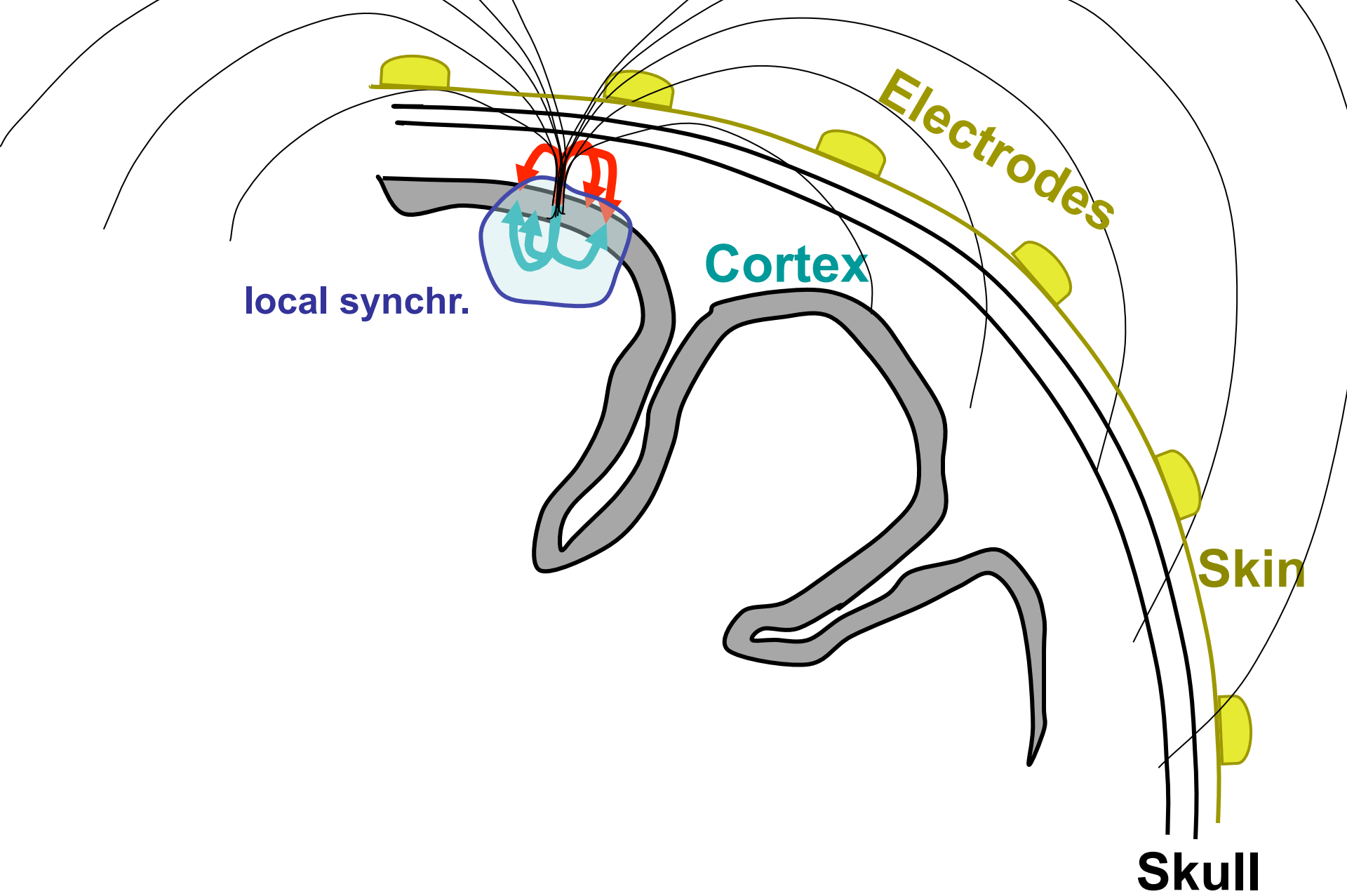
Electrodes

Skin

relative independence

local synchr.

Skull



# Source modelling of independent components

ICA takes care of unmixing of timeseries

Source analysis to take care of the location

Assumption: components correspond to compact spatial patches (or bilateral patches)

Use simple dipole models to model the spatial component topographies

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## Forward modelling

Required for the interpretation of scalp topographies

Different methods with varying accuracy

## Inverse modelling

Estimate 1) location and 2) timecourse

## Assumptions on source locations

Single or multiple point-like source

Distributed source

## Assumptions on source timecourse

Uncorrelated (and dipolar)

Independent



# Summary 2

Independent component analysis

separates topography and timecourse

Inverse methods to interpret topography

Single or multiple point-like source

Distributed source