M/EEG toolkit, Nijmegen, April 20, 2016

Connectivity analysis of electrophysiological data

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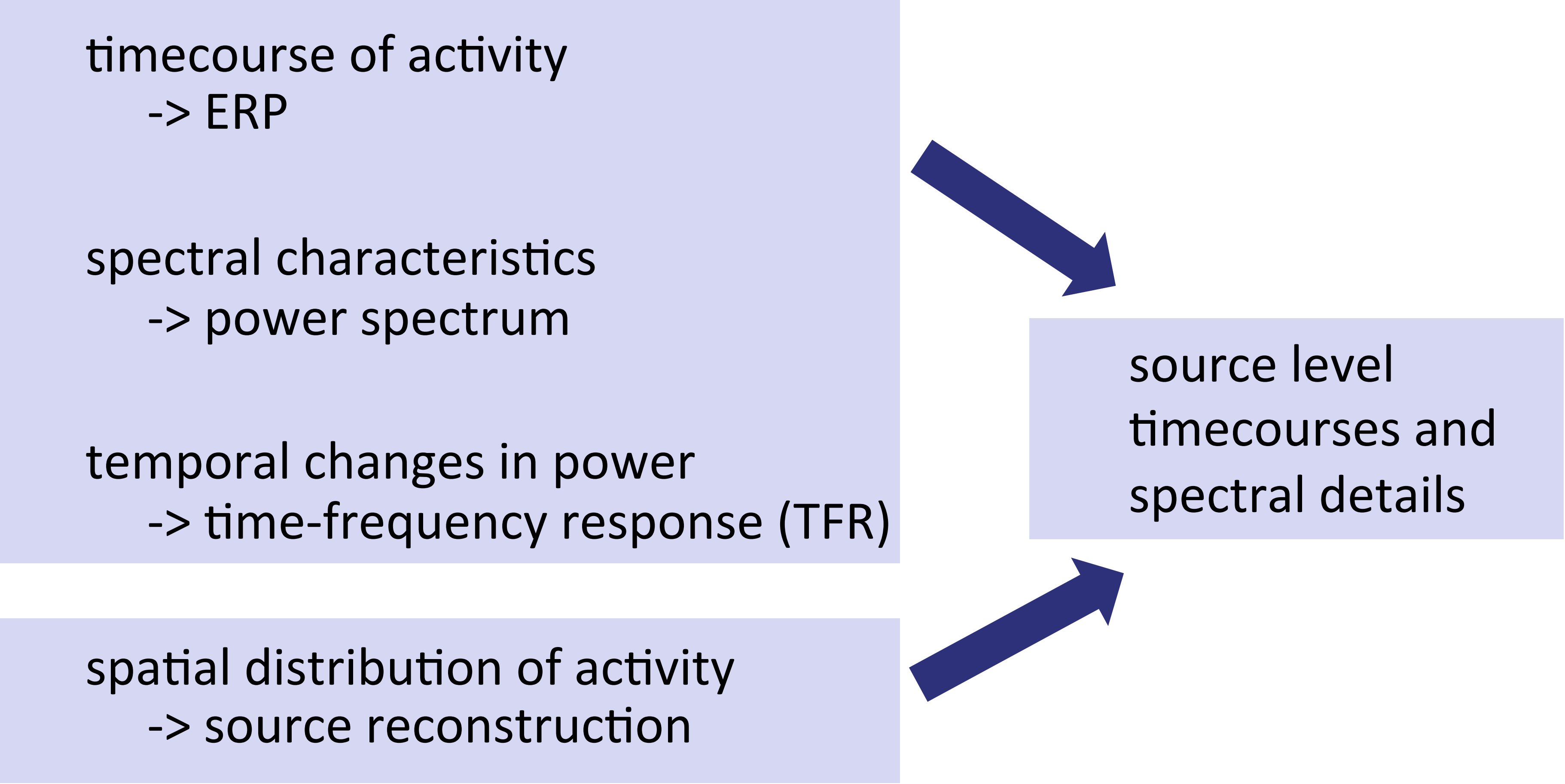
M/EEG signal characteristics considered during analysis



timecourse of activity
-> ERP

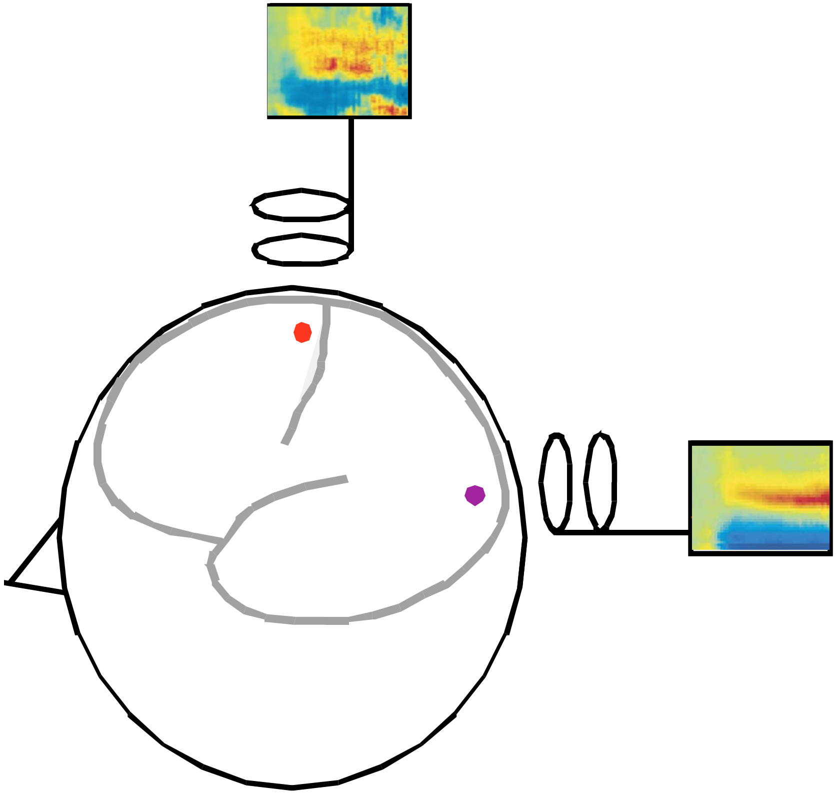
spectral characteristics
-> power spectrum

temporal changes in power
-> time-frequency response (TFR)

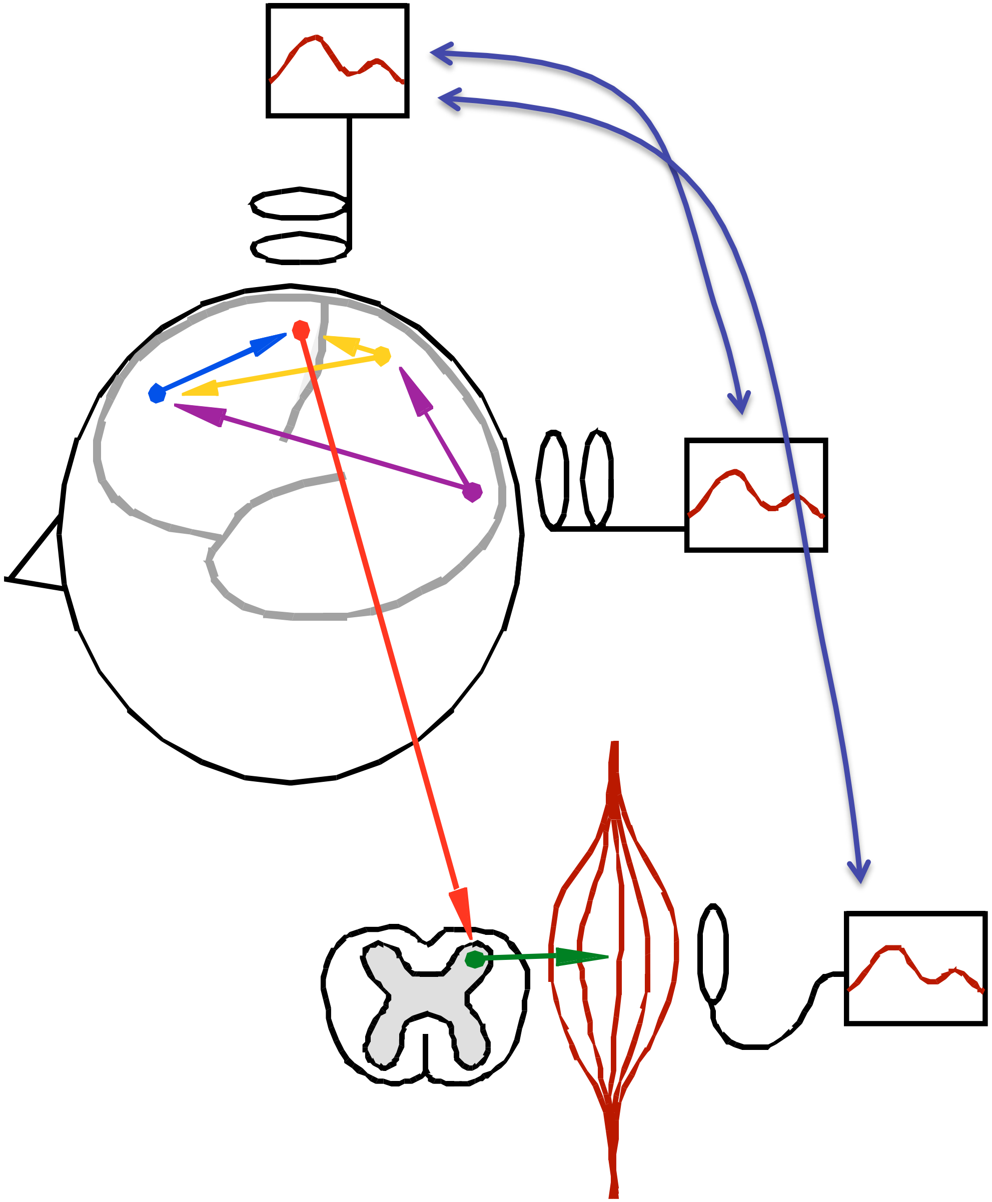
spatial distribution of activity
-> source reconstruction

source level timecourses and spectral details

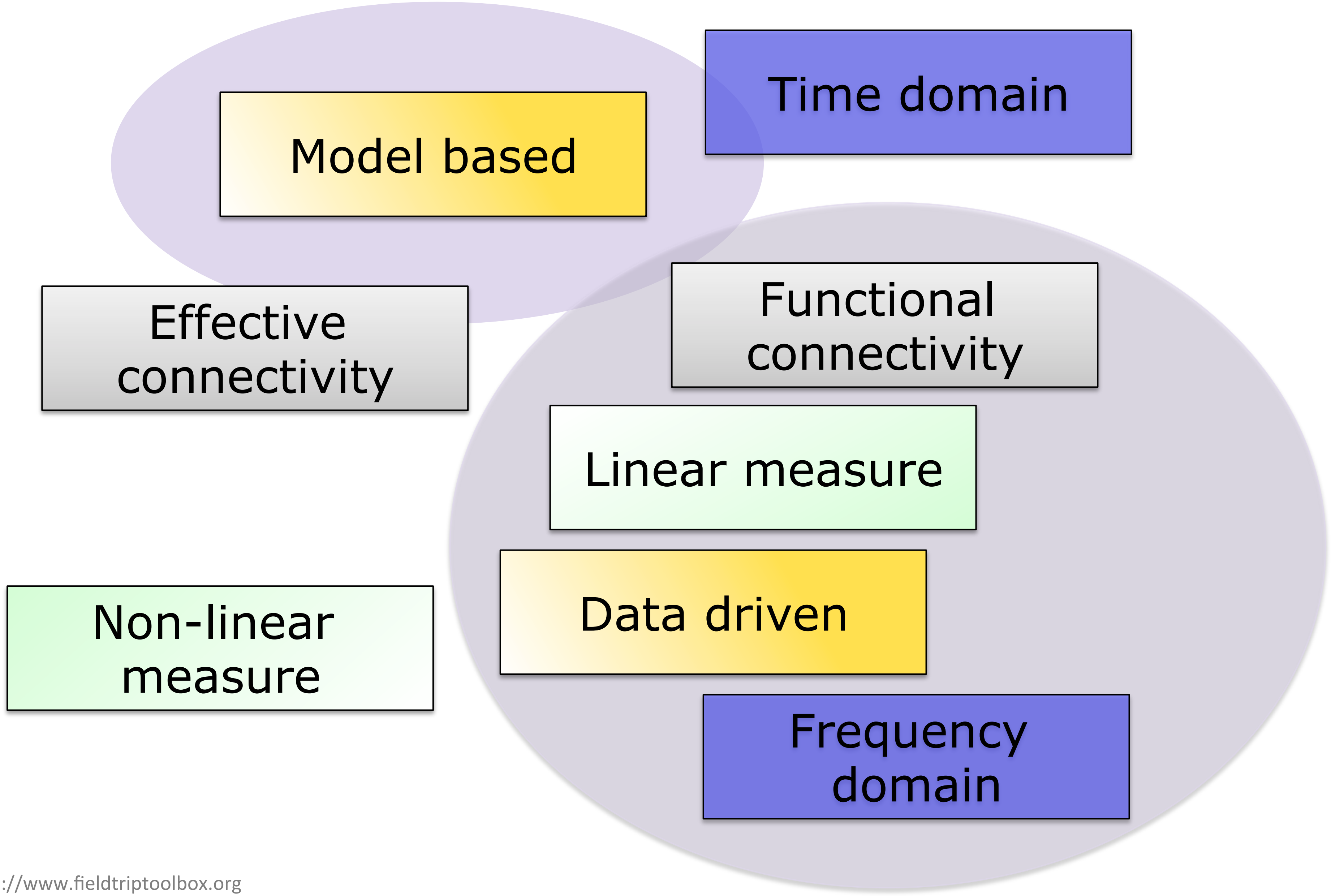
Univariate analysis



Connectivity analysis: Beyond univariate analysis



Measures of connectivity



Model based

Time domain

Effective connectivity

Functional connectivity

Linear measure

Non-linear measure

Data driven

Frequency domain

Measures of frequency domain connectivity

Coherence coefficient

Directed transfer function

Phase locking value

Phase lag index

Phase synchronization

Imaginary part of coherency

Partial directed coherence

Pairwise phase consistency

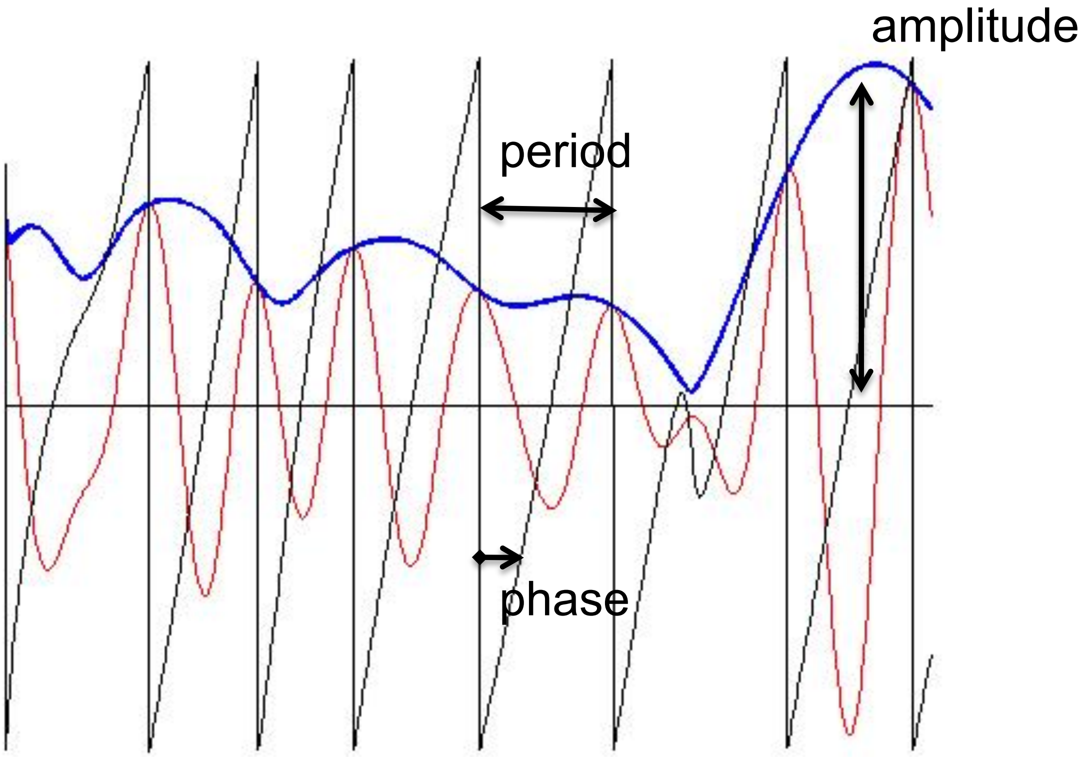
Phase slope index

Synchronization likelihood

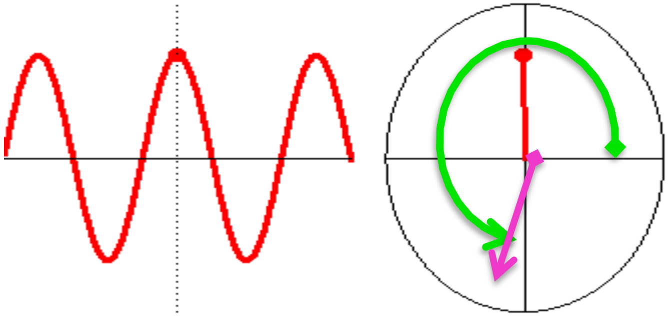
Frequency domain granger causality

Measures of frequency domain connectivity

What constitutes an oscillation? (recap)



What constitutes an oscillation? (the movie)



$$x = Ae^{i\varphi}$$

What about 2 oscillations?
Let’s look at the phase difference

phase signal 1

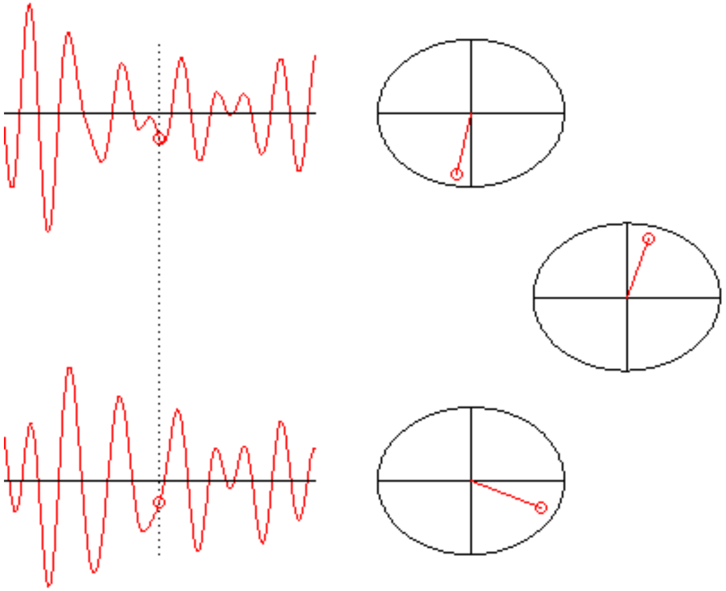
phase difference

phase signal 2

Phase difference is scattered:
Low synchrony

What about 2 oscillations?
Let’s look at the phase difference

$x_1 = A_1 e^{i\varphi_1}$



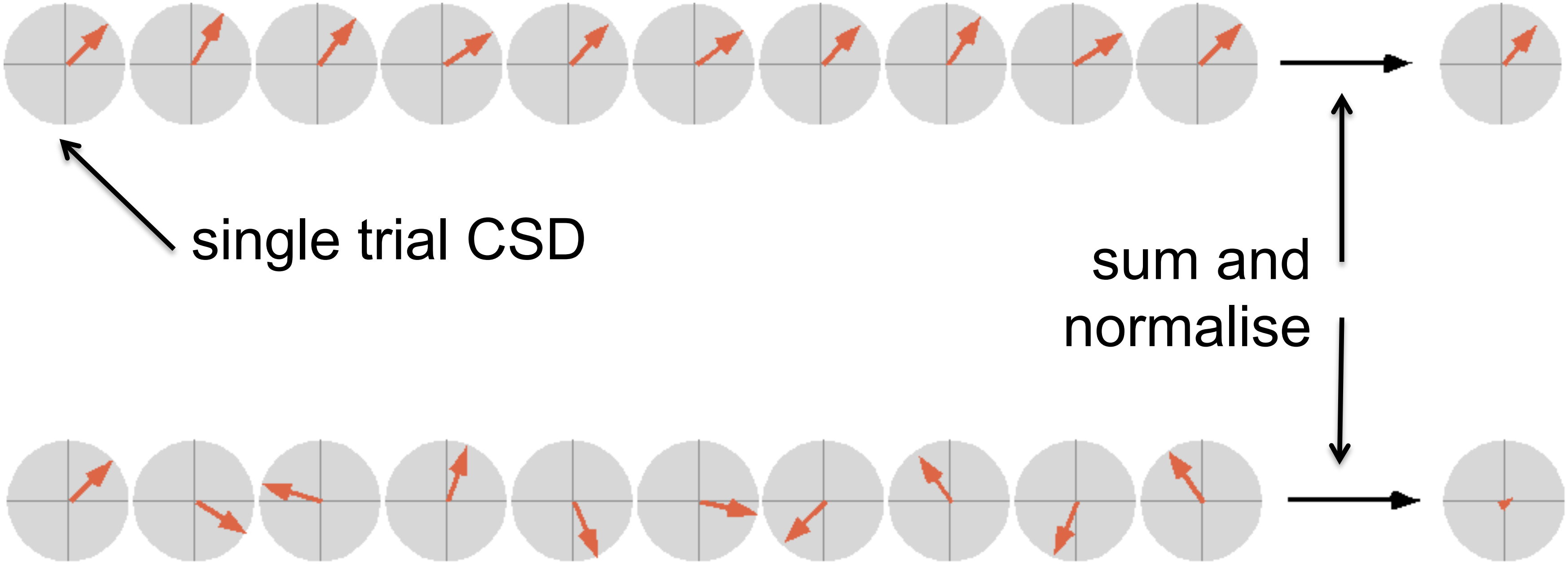
$x_2 = A_2 e^{i\varphi_2}$

Phase difference is clustered:
High synchrony

Measures of connectivity: coherence (the math view)

Coherence is computed from the *cross-spectral density*, which is obtained by *conjugate multiplication* of the frequency domain representation of the signals

$$x_1x_2^* = A_1e^{i\varphi_1} \times A_2e^{-i\varphi_2} = A_1A_2e^{i(\varphi_1-\varphi_2)}$$



single trial CSD

sum and normalise

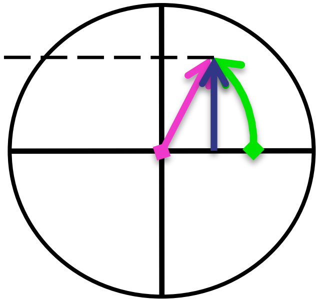
Measures of connectivity: coherence & co

$$\text{Coherence} = \left| \frac{1/N \sum A_1 A_2 e^{i(\varphi_1-\varphi_2)}}{\sqrt{(1/N \sum A_1^2)(1/N \sum A_2^2)}} \right|$$

$$\text{PLV} = \left| \frac{1/N \sum 1 \times 1 \times e^{i(\varphi_1-\varphi_2)}}{\sqrt{(1/N \sum 1^2)(1/N \sum 1^2)}} \right| = \left| \frac{\sum e^{i(\varphi_1-\varphi_2)}}{N} \right|$$

Measures of connectivity: coherence & co

$$\text{Coherency} = \frac{1/N \, \Sigma A_1 A_2 e^{i(\varphi_1 - \varphi_2)}}{\sqrt{(1/N \, \Sigma A_1^2)(1/N \, \Sigma A_2^2)}} = C e^{i\Delta\varphi}$$

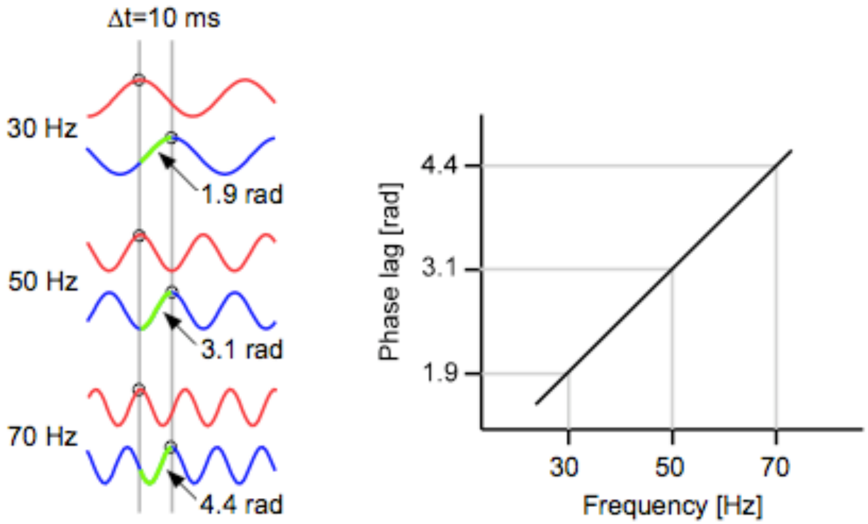


Imaginary part of coherency

Measures of connectivity: coherence & co

$$\text{Coherency} = \frac{1/N\ \Sigma A_1 A_2 e^{i(\varphi_1-\varphi_2)}}{\sqrt{(1/N\ \Sigma A_1^2)(1/N\ \Sigma A_2^2)}} = Ce^{i\Delta\varphi}$$

Slope of relative phase spectrum indicates time delay



http://www.fieldtriptoolbox.org

Coherence and linear prediction

Coherence coefficient ~ cross-correlation coefficient

|Coherence|^2 ~ % variance explained

Coherence coefficient similar to frequency domain regression

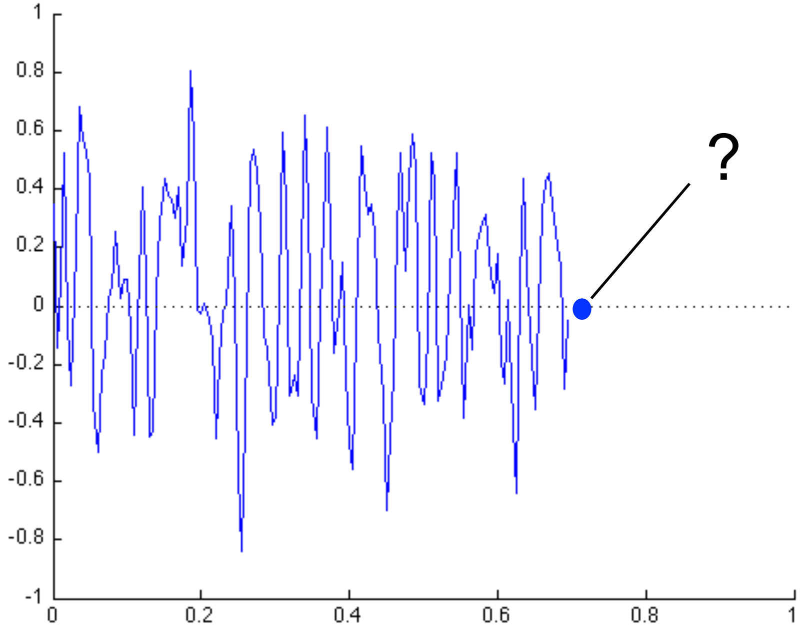
Conceptual difference with regression: independent and dependent variable are interchangeable

Slope of relative phase spectrum indicates the temporal precedence (~ directed influence)

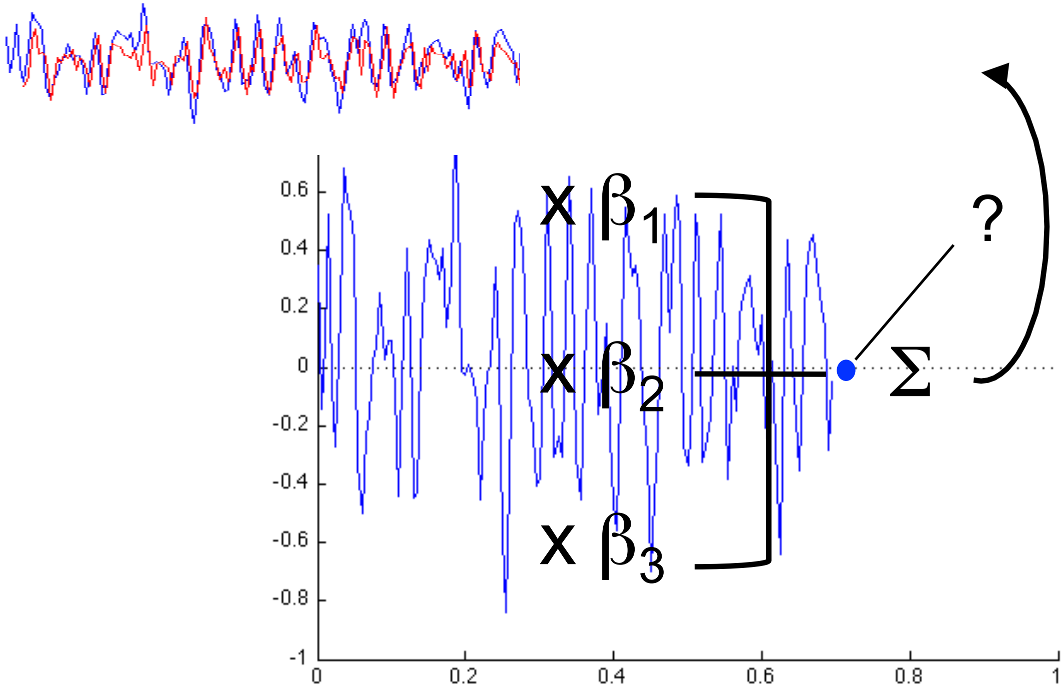
Slope often hard to estimate or close to zero

Linear prediction and directed interaction: the concept of Granger causality

Linear prediction and directed interaction: the concept of Granger causality

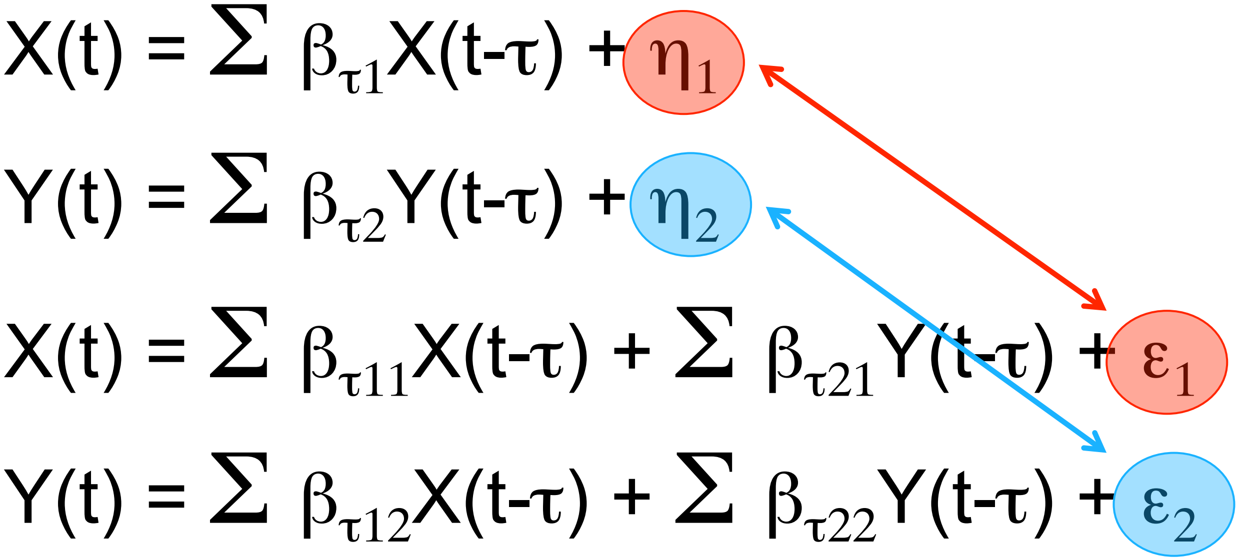


Linear prediction: autoregressive models



$$X(t) = \sum \beta_\tau X(t-\tau) + \eta$$

Two signals: bivariate autoregressive models



$$X(t) = \sum \beta_{\tau 1} X(t-\tau) + \eta_1$$

$$Y(t) = \sum \beta_{\tau 2} Y(t-\tau) + \eta_2$$

$$X(t) = \sum \beta_{\tau 11} X(t-\tau) + \sum \beta_{\tau 21} Y(t-\tau) + \varepsilon_1$$

$$Y(t) = \sum \beta_{\tau 12} X(t-\tau) + \sum \beta_{\tau 22} Y(t-\tau) + \varepsilon_2$$

Granger causality: compare the residuals

$$X(t) = \sum \beta_{\tau 1} X(t-\tau) + \eta_1$$

$$Y(t) = \sum \beta_{\tau 2} Y(t-\tau) + \eta_2$$

$$X(t) = \sum \beta_{\tau 11} X(t-\tau) + \sum \beta_{\tau 21} Y(t-\tau) + \varepsilon_1$$

$$Y(t) = \sum \beta_{\tau 12} X(t-\tau) + \sum \beta_{\tau 22} Y(t-\tau) + \varepsilon_2$$

$$F_{Y \rightarrow X} = \ln\left(\frac{\mathrm{var}(\eta_1)}{\mathrm{var}(\varepsilon_1)}\right)$$

$$F_{X \rightarrow Y} = \ln\left(\frac{\mathrm{var}(\eta_2)}{\mathrm{var}(\varepsilon_2)}\right)$$

Analogy between Granger and 'plain' regression

$X(t) = \sum \beta_{\tau 1} X(t-\tau) + \eta_1$

$Y(t) = \sum \beta_{\tau 2} Y(t-\tau) + \eta_2$

$X(t) = \sum \beta_{\tau 11} X(t-\tau) + \sum \beta_{\tau 21} Y(t-\tau) + \varepsilon_1$

$Y(t) = \sum \beta_{\tau 12} X(t-\tau) + \sum \beta_{\tau 22} Y(t-\tau) + \varepsilon_2$

$\text{data} = \sum \beta_\kappa X_\kappa + \eta$

$\text{data} = \sum \beta'_{\kappa} X_\kappa + \beta'_{\kappa+1} X_{\kappa+1} + \varepsilon$

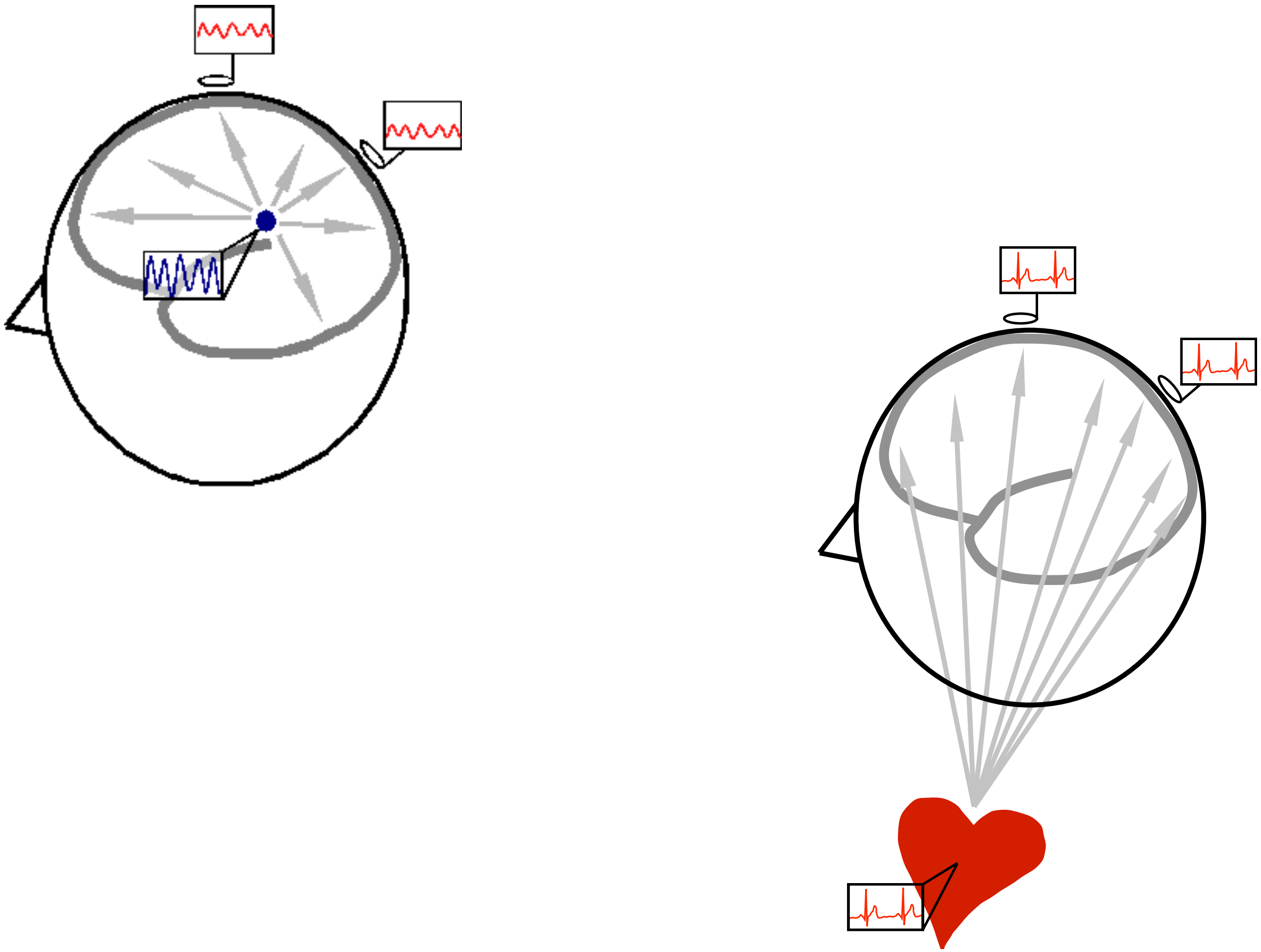
$$F_{Y \rightarrow X} = \ln\left(\frac{\mathrm{var}(\eta_1)}{\mathrm{var}(\varepsilon_1)}\right)$$

$$F \sim \frac{\mathrm{var}(\eta)}{\mathrm{var}(\varepsilon)}$$

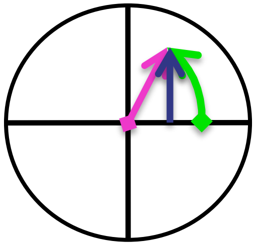
…only the inference is different

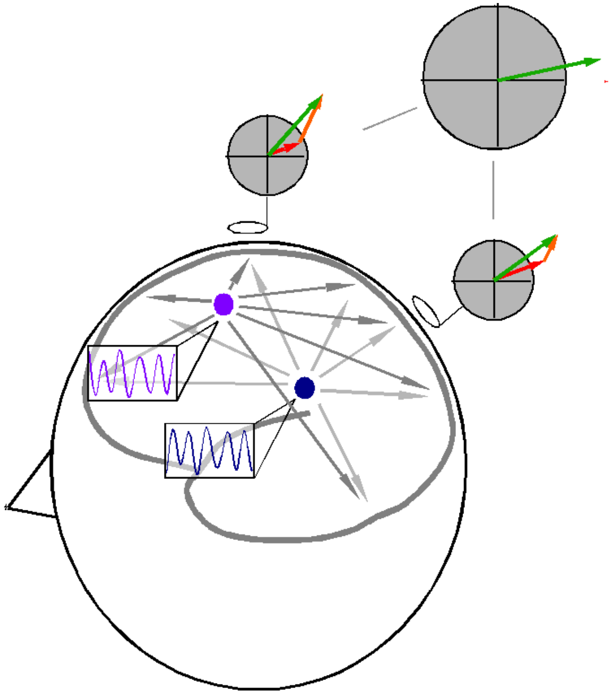
MEG connectivity
…
implementation

Practial issues: Electromagnetic field spread



Practical issues: imaginary part of coherency

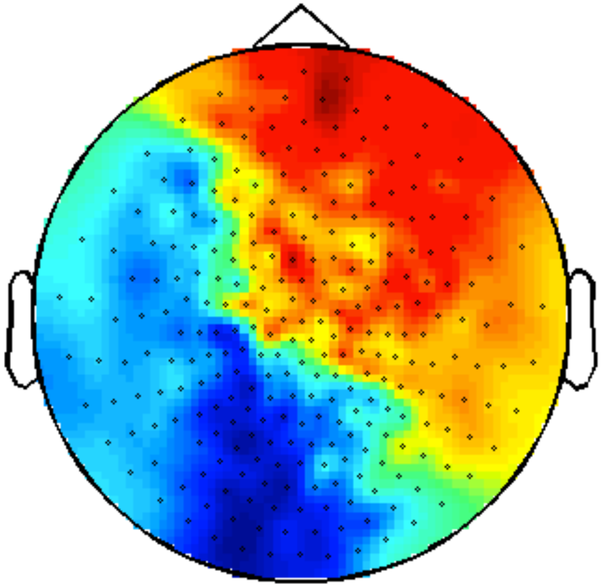


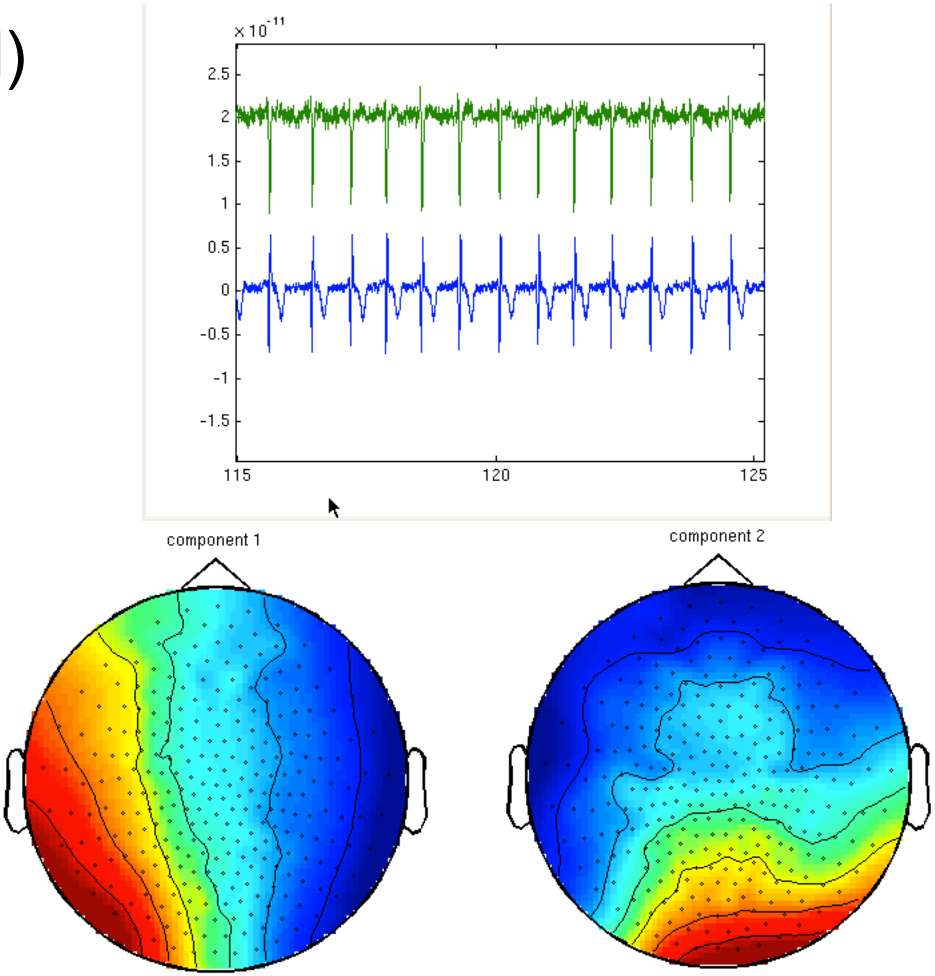


Im(coherency) ≠ **0**

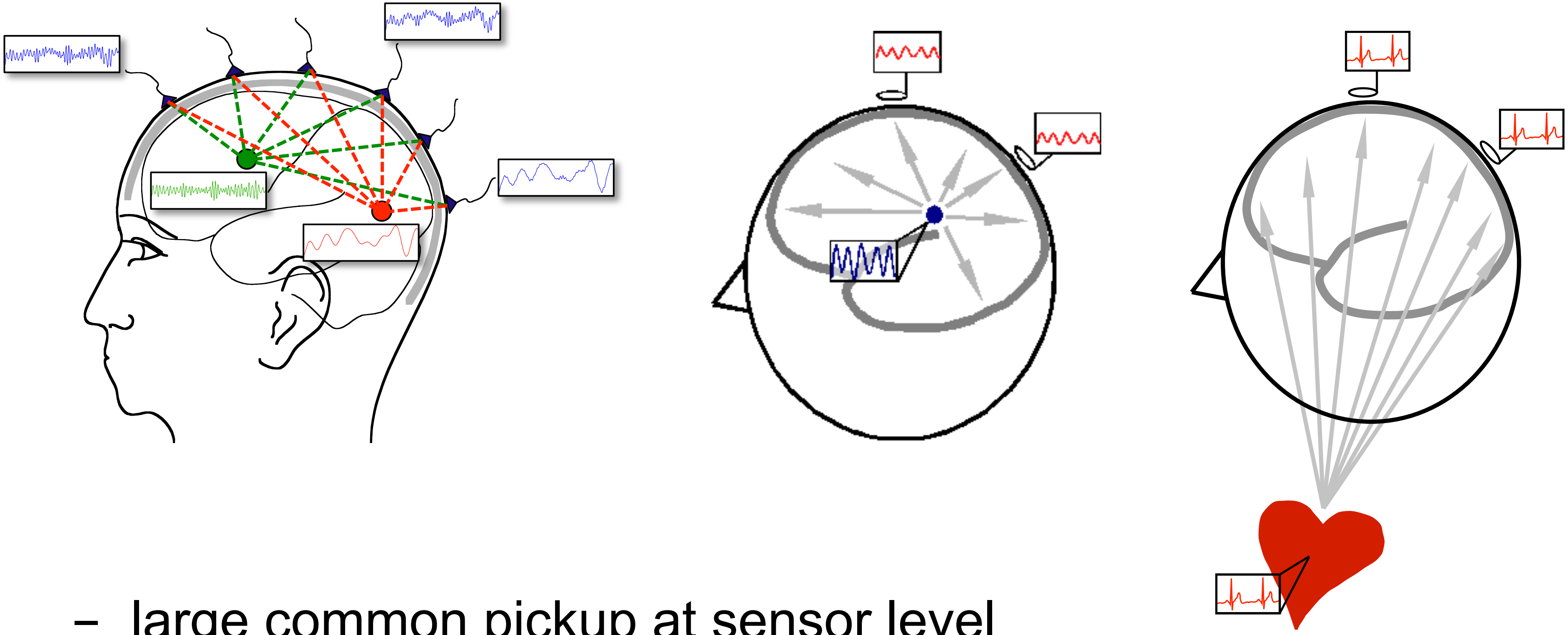
MEG connectivity: pitfalls with assumptions

WPLI suggests fronto-occipital directed interaction (alpha band)





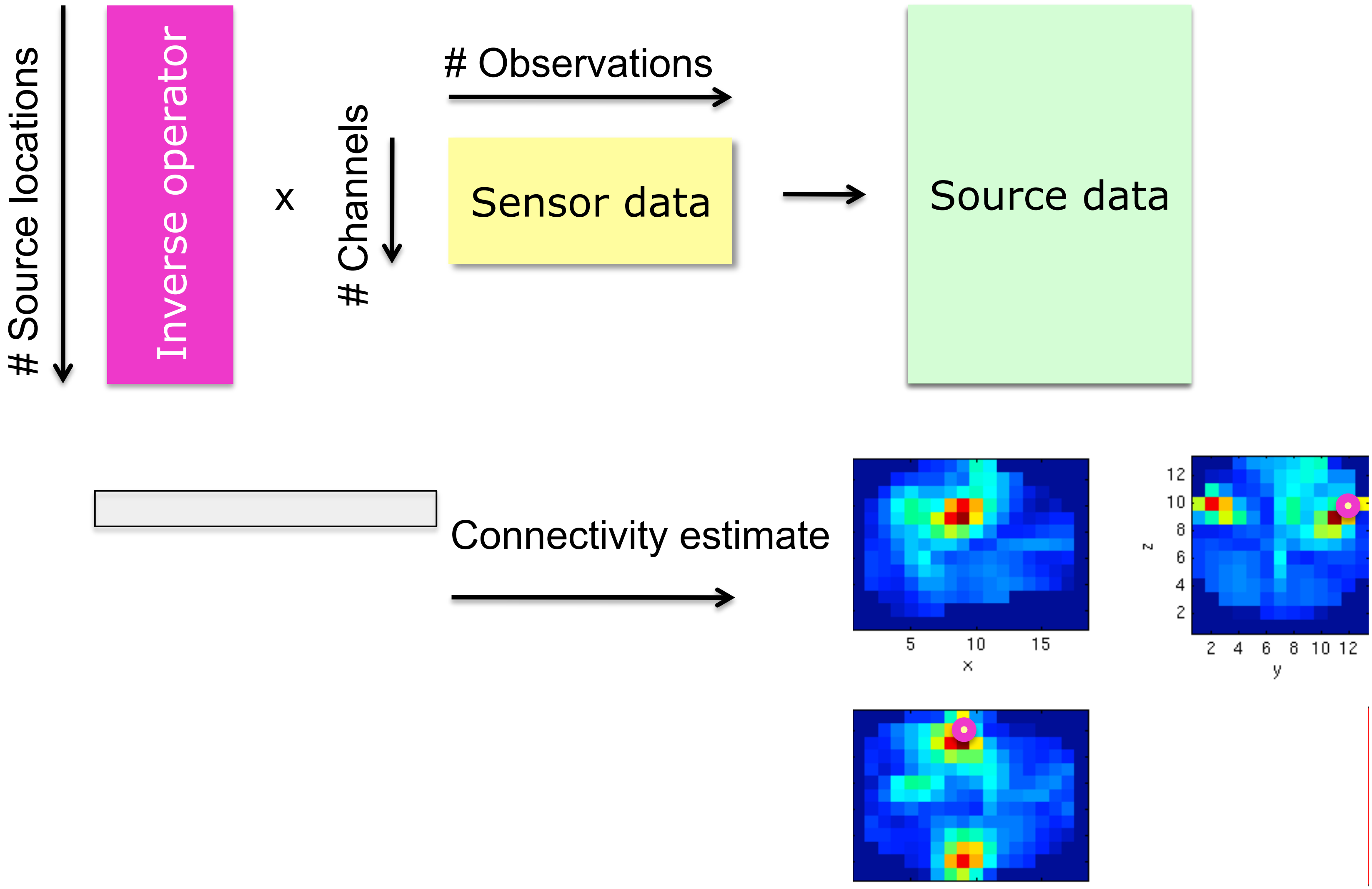
Common pick up



- large common pickup at sensor level
- not all interfering sources are 1-dimensional
- no common pickup if you have a perfect source model
- some common pickup if source model is not perfect

Better to do source reconstruction first

Compute connectivity at the source level



Source locations

Inverse operator

x

Channels

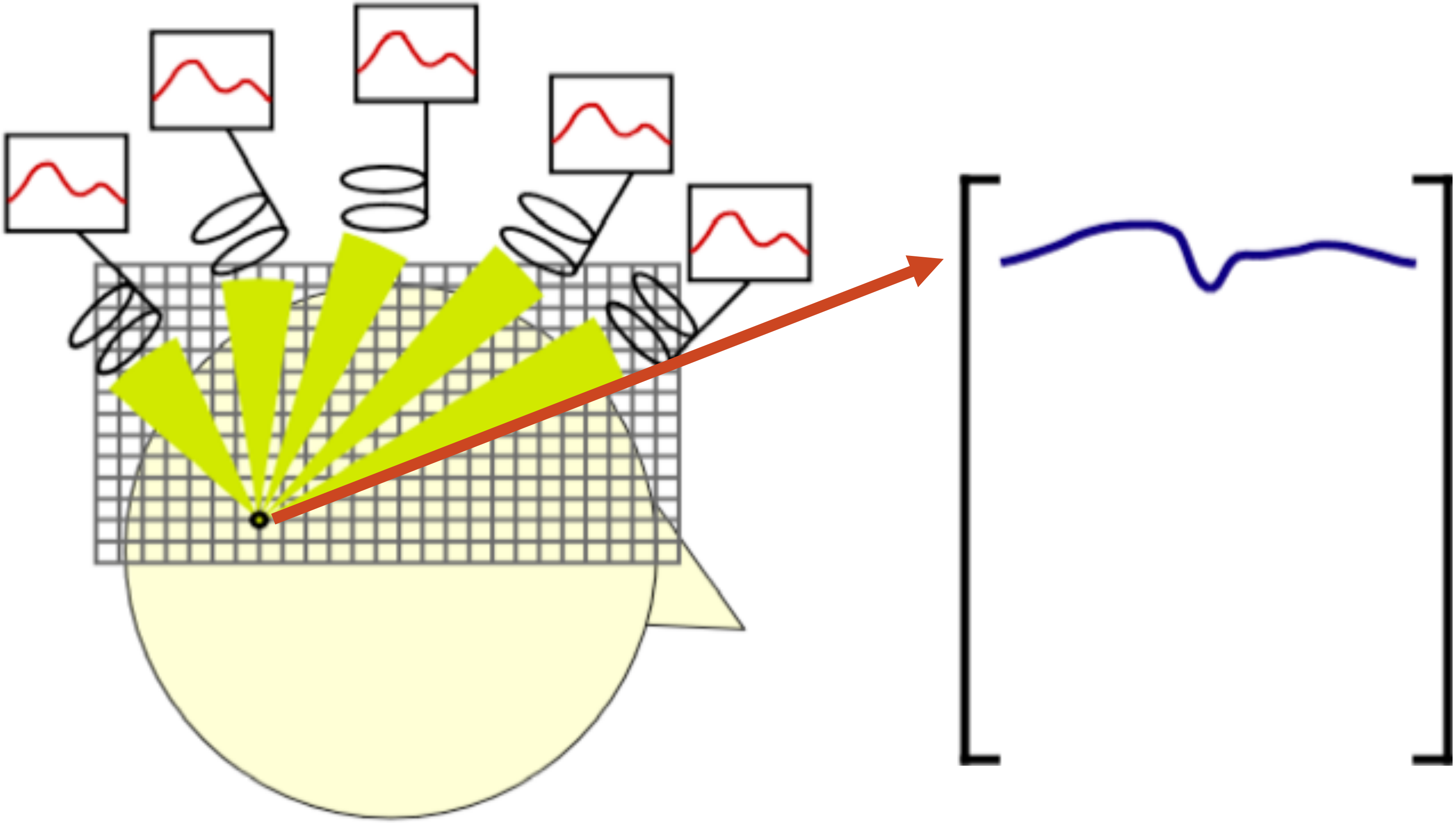
Observations

Sensor data

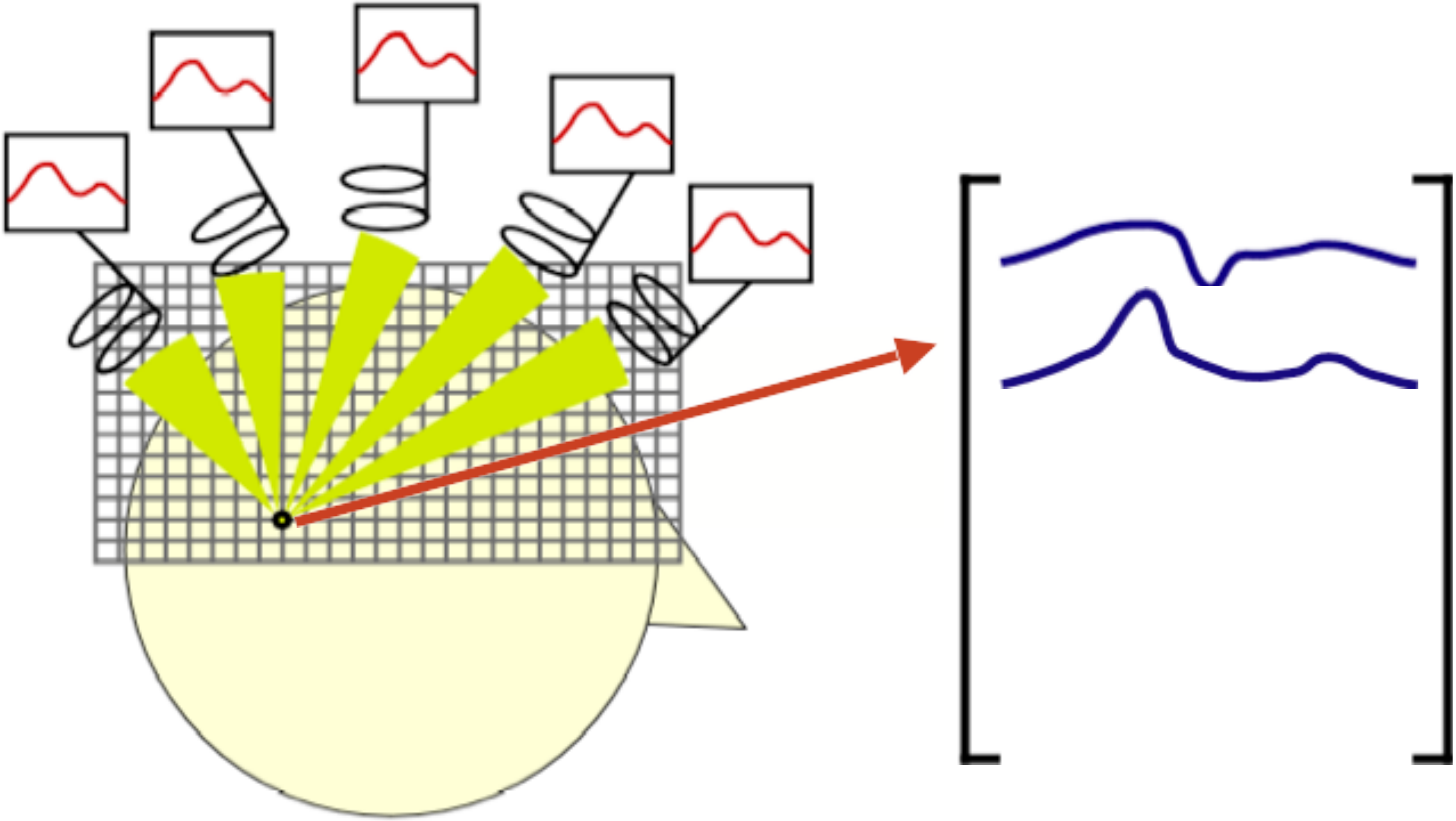
Source data

Connectivity estimate

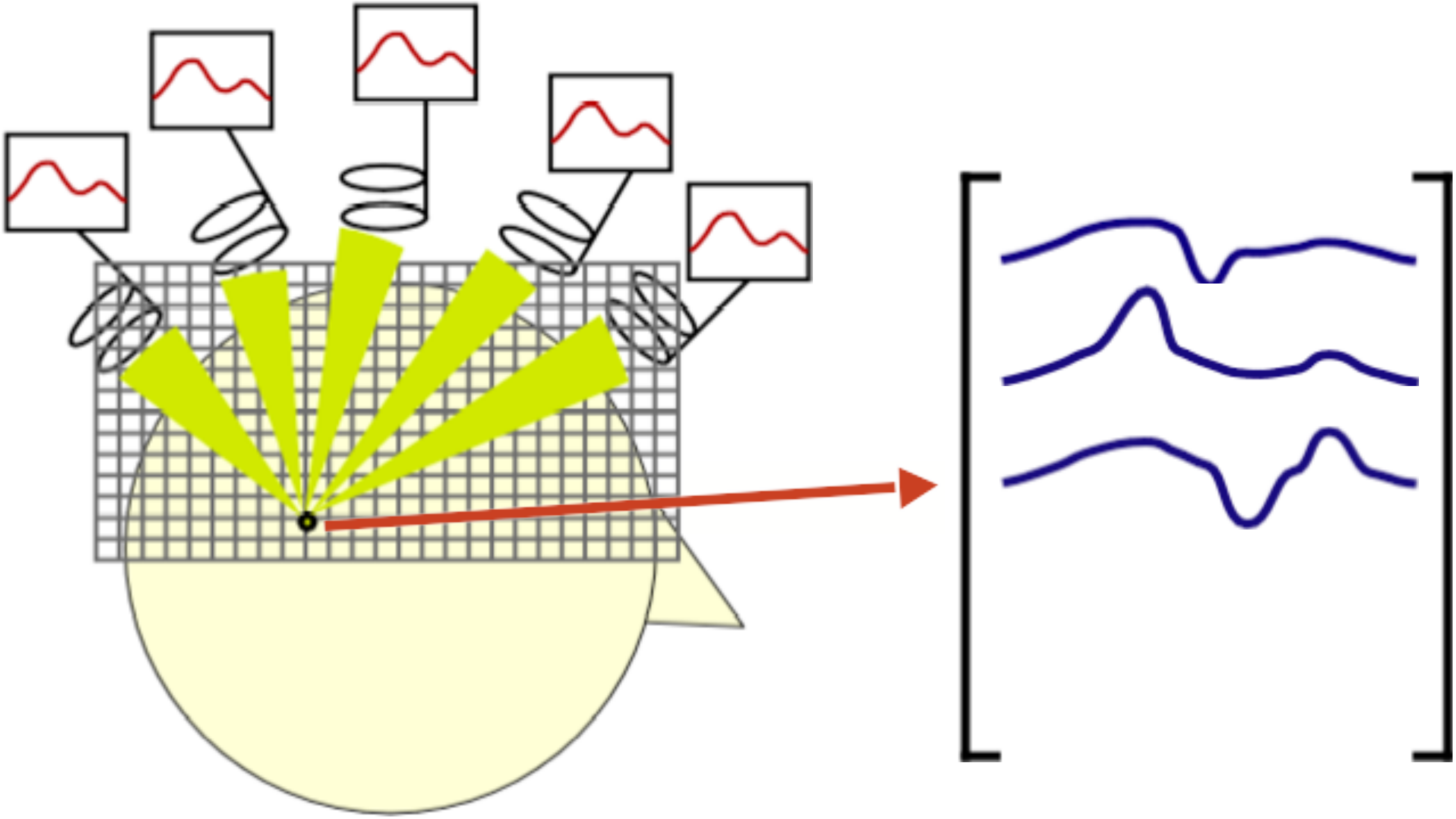
Beamformers: the concept



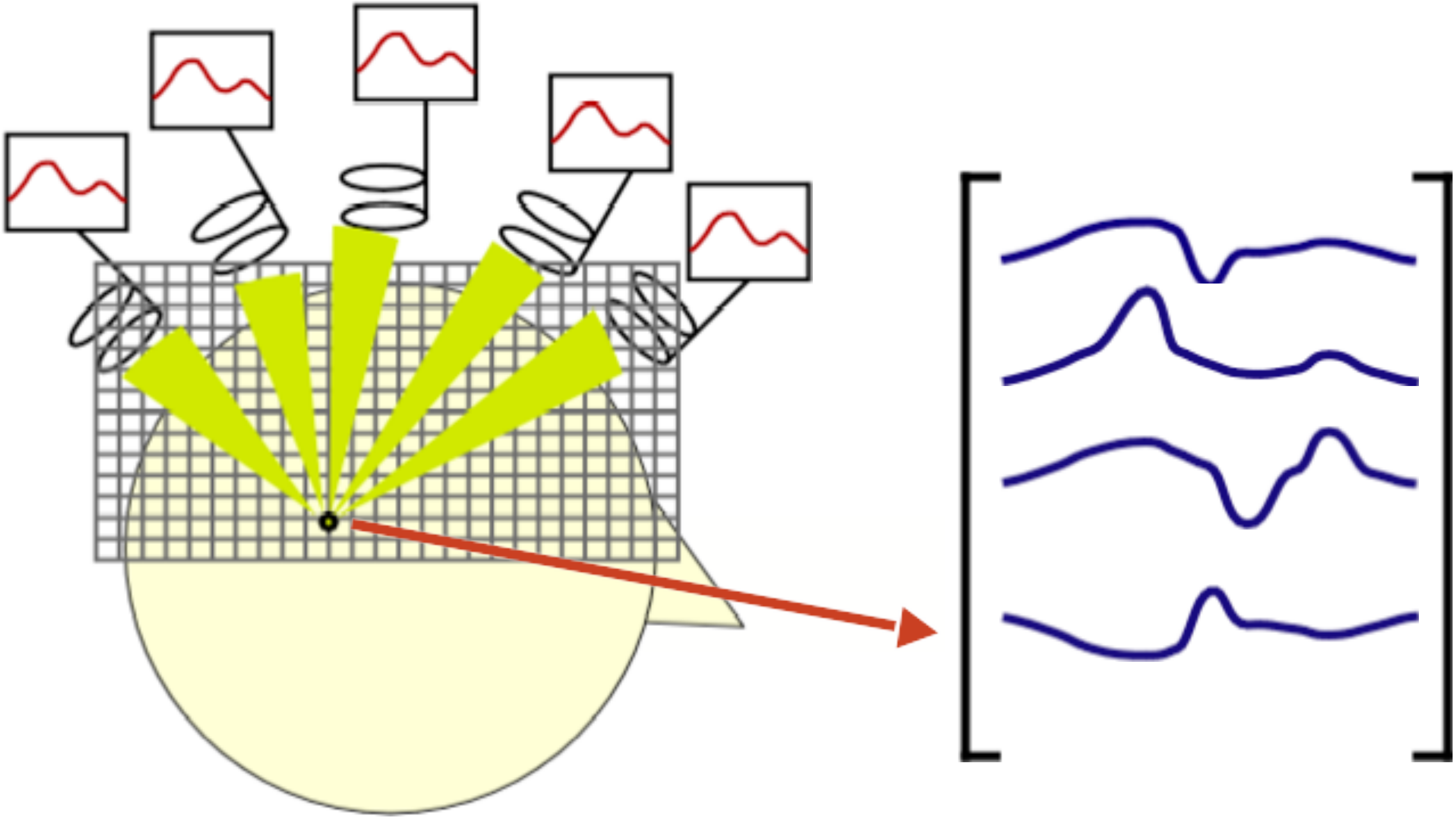
Beamformers: the concept



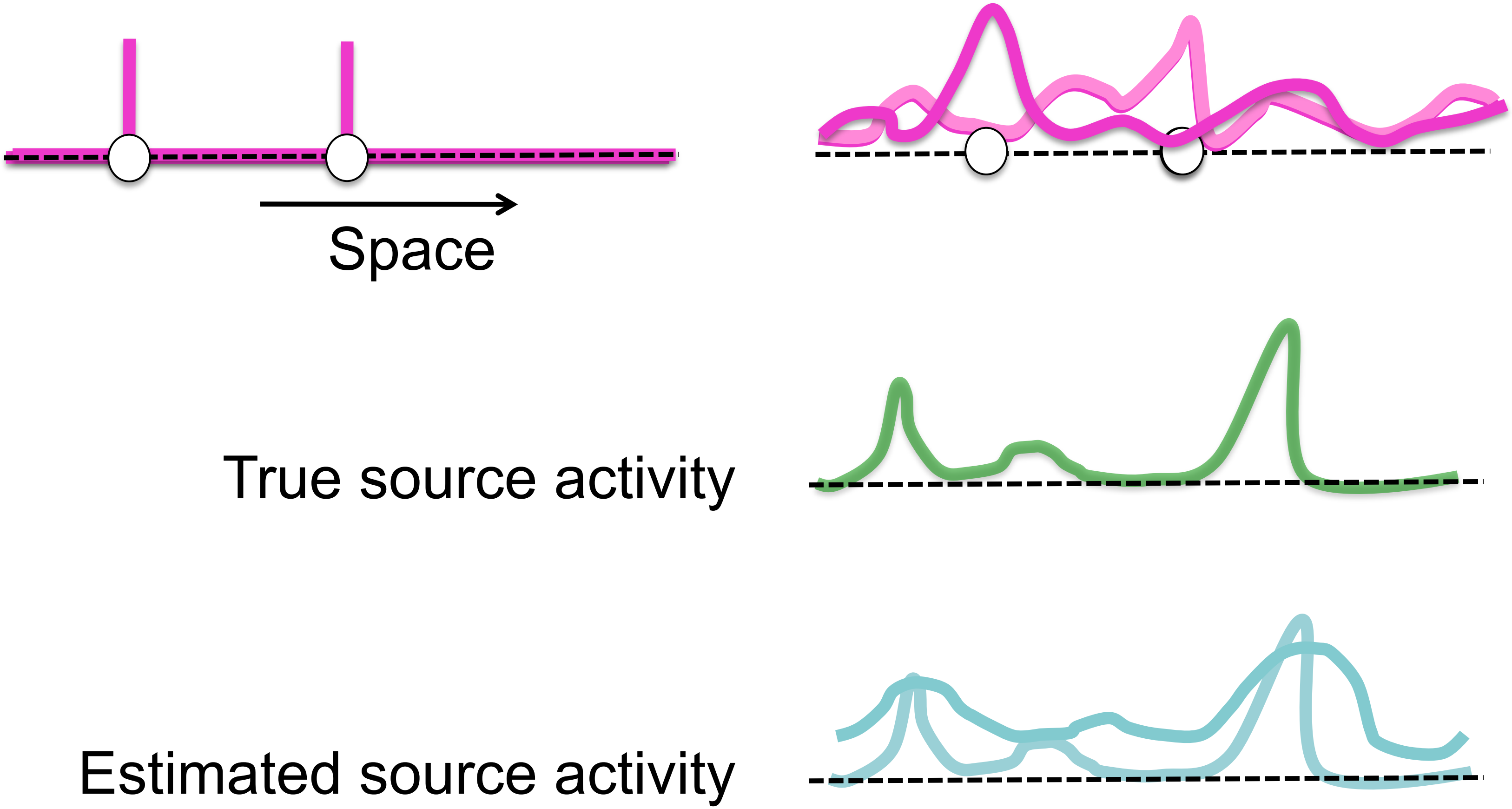
Beamformers: the concept



Beamformers: the concept



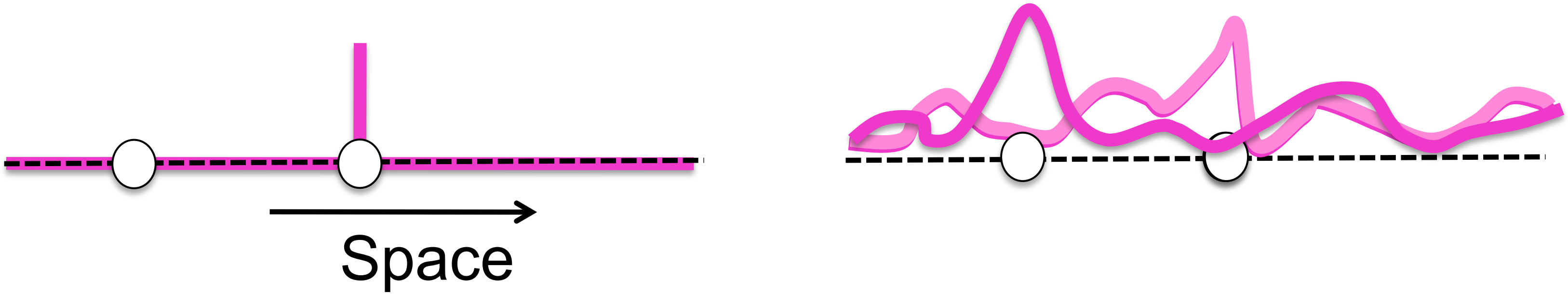
Features of spatial filters



True source activity

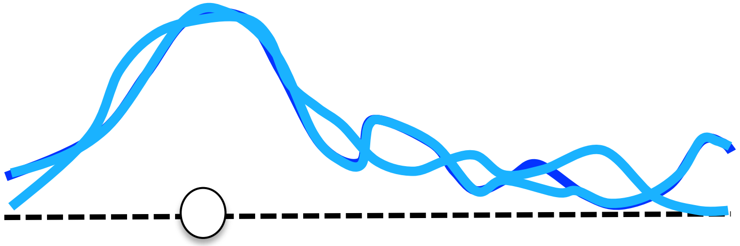
Estimated source activity

Features of spatial filters:
spurious connectivity due to spatial leakage of ‘noise’



True source connectivity

Estimated source conn



Confounds for connectivity

Common pick up

- other sources in the brain
- other physiological sources
- especially problematic if those sources have some “internal synchronization” themselves

Differences in signal (or noise) between experimental conditions

- better SNR -> more reliable estimate of the phase
- more reliable phase -> more conistent phase difference

Concluding remarks

Connectivity analysis is cool

Many measures on the market

Many of the confounds are not easy to deal with

Interpretation of results should therefore be done with care