

Forward and Inverse Modeling of EEG and MEG data

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Motivation and background

Forward modeling

Source model

Volume conductor model

Inverse modeling

Single and multiple dipole fitting

Distributed source models

Beamforming methods

Summary



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Motivation



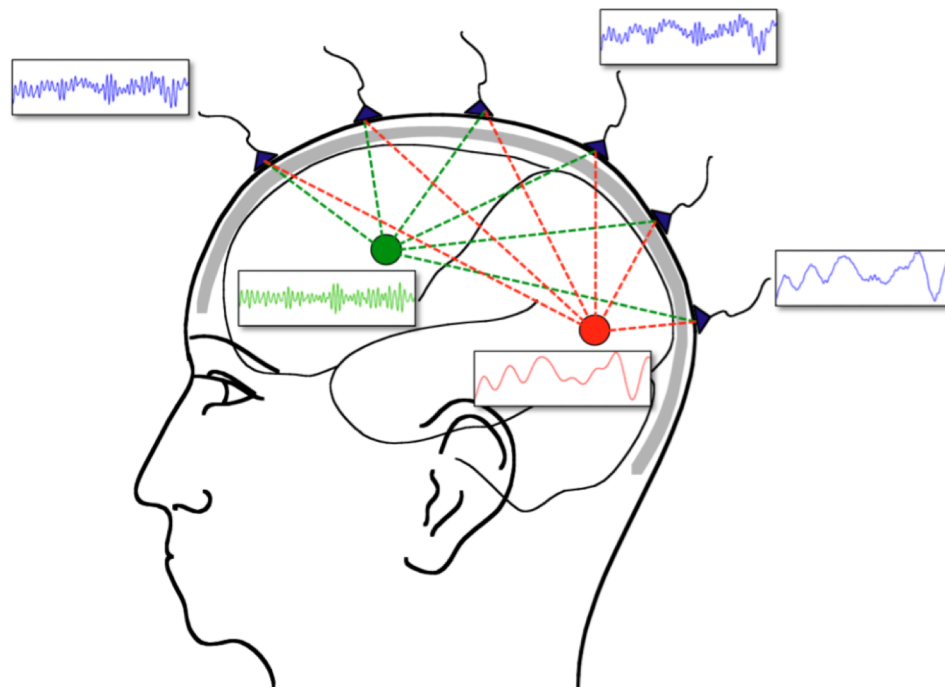
- Strong points of EEG and MEG
 - Temporal resolution (~ 1 ms)
 - Characterize individual components of ERP/ERF
 - Oscillatory activity
 - Disentangle dynamics of cortical networks
- Weak points of EEG and MEG
 - Measurement on outside of the brain
 - Overlap of components
 - Low spatial resolution

Motivation

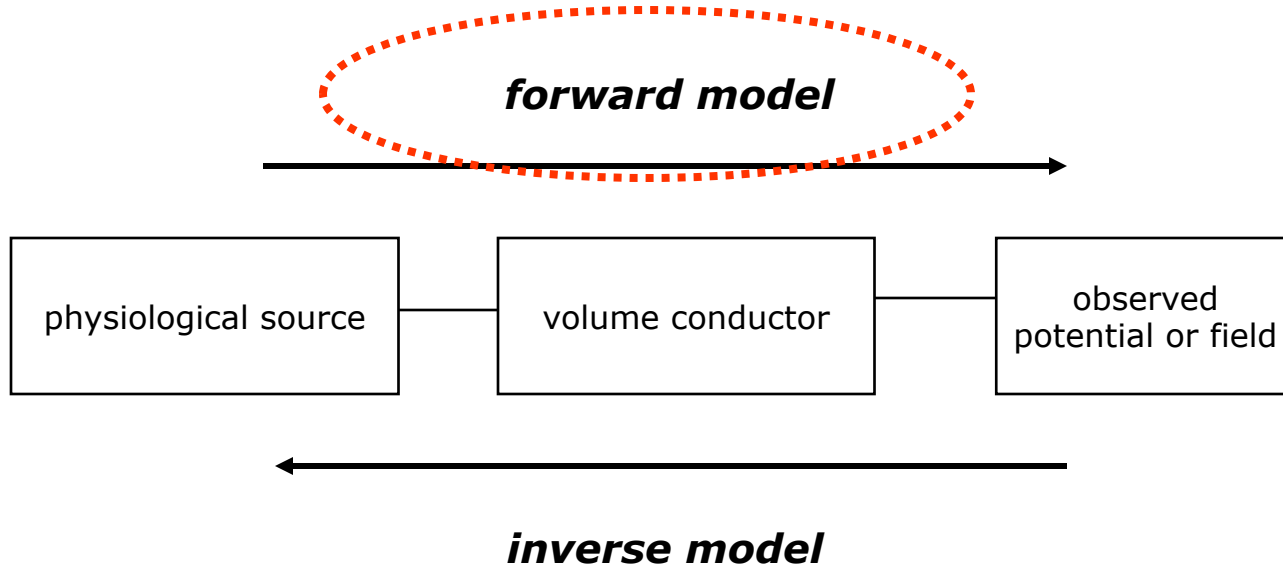


- If you find a ERP/ERF component, you want to characterize it in physiological terms
 - Time or frequency are the “natural” characteristics
 - “Location” requires interpretation of the scalp topography
- Forward and inverse modeling helps to interpret the topography
- Forward and inverse modeling helps to disentangle overlapping source timeseries

Superposition of source activity



Biophysical source modelling: overview





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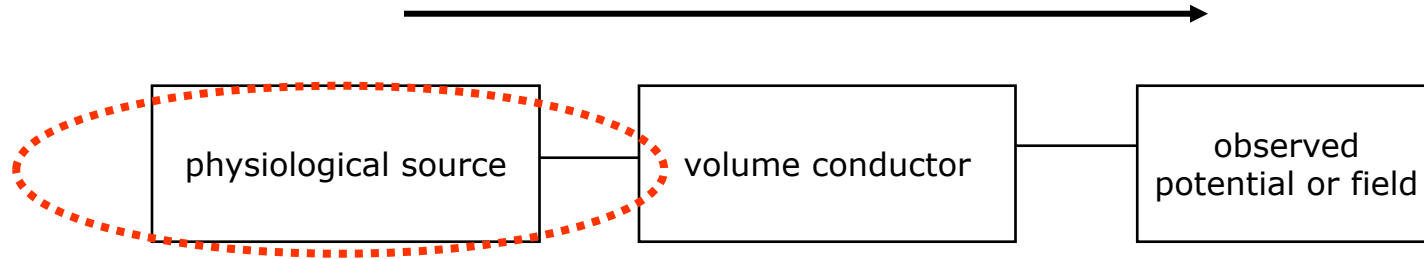
Beamforming methods

Summary

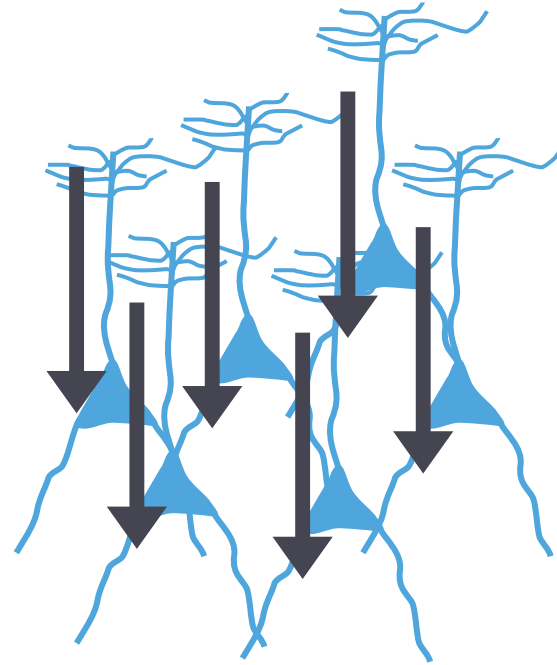
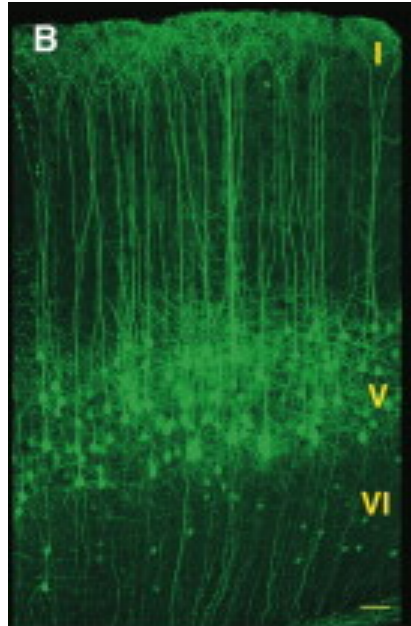
Biophysical source modelling: overview



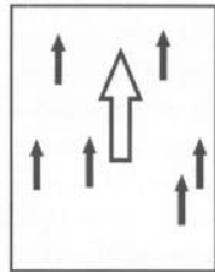
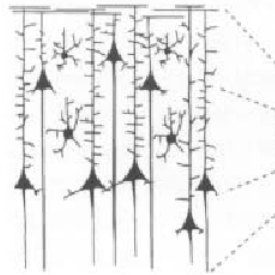
forward model



What produces the electric current



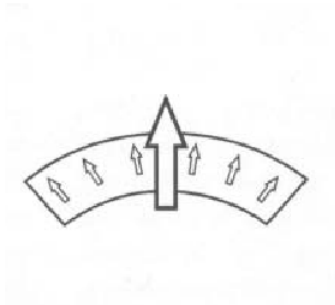
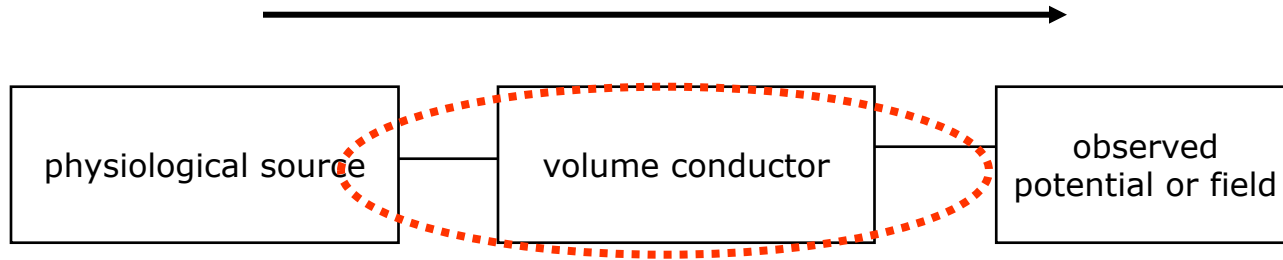
Equivalent current dipoles



Biophysical source modelling: overview

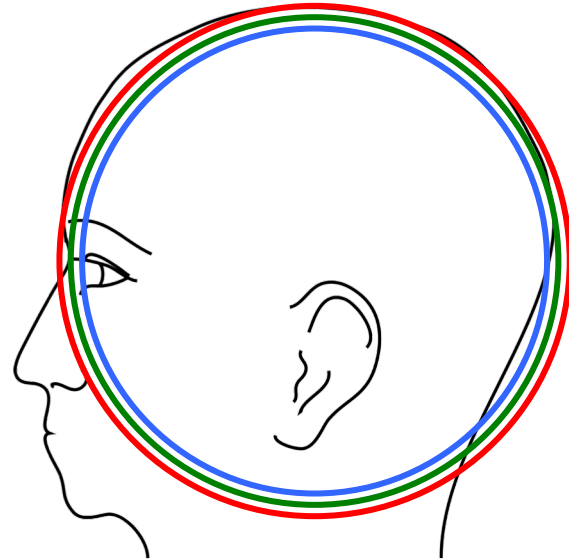


forward model



Volume conductor

- described electrical properties of tissue
- describes geometrical model of the head
- describes **how** the currents flow, not where they originate from
- same volume conductor for EEG as for MEG, but also for tDCS, tACS, TMS, ...



Volume conductor

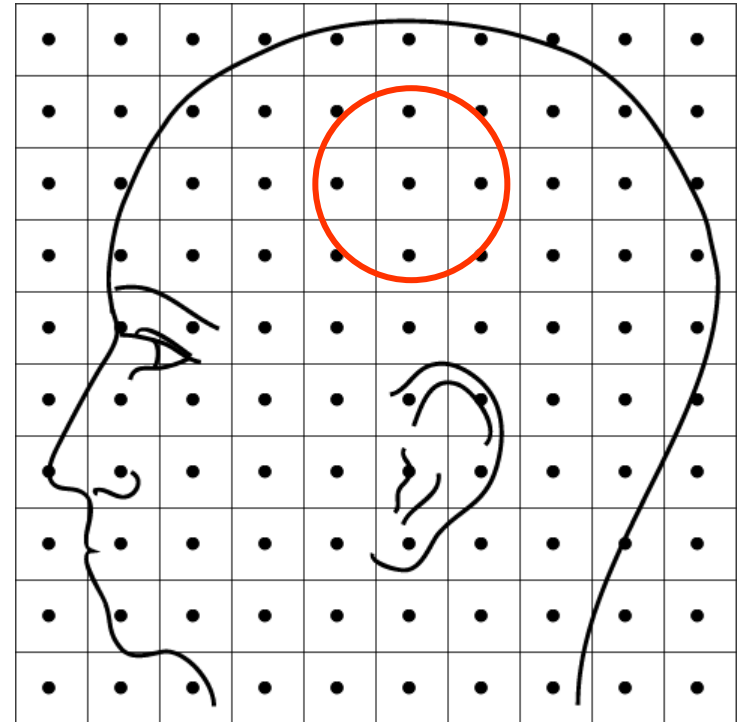


- Computational methods for volume conduction problem that allow for **realistic geometries**
 - FDM *Finite Difference Method*
 - BEM *Boundary Element Method*
 - FEM *Finite Element Method*

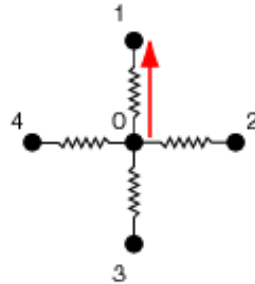
Volume conductor: Finite Difference Method



- Easy to compute
- Not very useful in practice



Volume conductor: Finite Difference Method



$$\left. \begin{array}{l} \text{Kirchhoffs law} \quad I_1 + I_2 + I_3 + I_4 = 0 \\ \text{Ohm's law} \quad V = I * R \end{array} \right\} \Rightarrow$$

$$\Delta V_1 / R_1 + \Delta V_2 / R_2 + \Delta V_3 / R_3 + \Delta V_4 / R_4 = 0 \quad \Rightarrow$$

$$(V_1 - V_0) / R_1 + (V_2 - V_0) / R_2 + (V_3 - V_0) / R_3 + (V_4 - V_0) / R_4 = 0$$



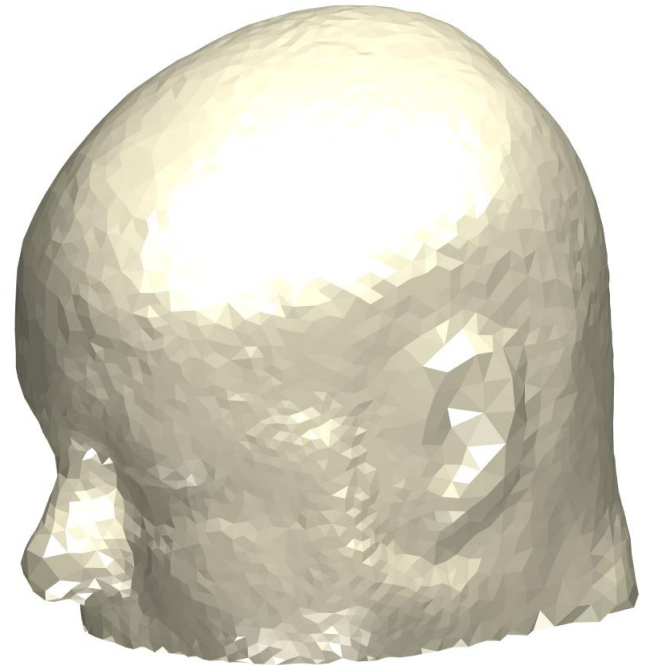
Volume conductor: Finite Difference Method

- Unknown potential V_i at each node
- Linear equation for each node
 - approx. $100 \times 100 \times 100 = 1.000.000$ linear equations
 - just as many unknown potentials
- Add a source/sink
 - sum of currents is zero for all nodes, except
 - sum of current is I_- for a certain node
 - sum of current is I_+ for another node
- Solve for unknown potential



Volume conductor: Boundary Element Method

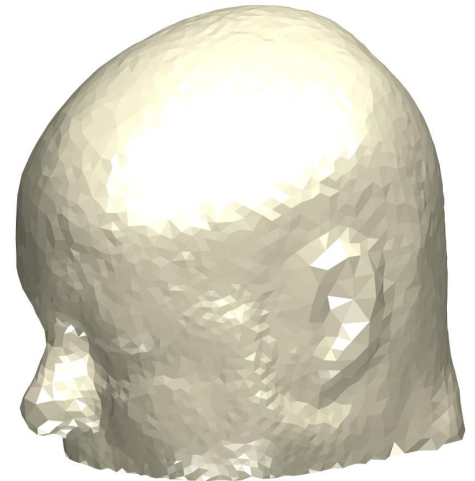
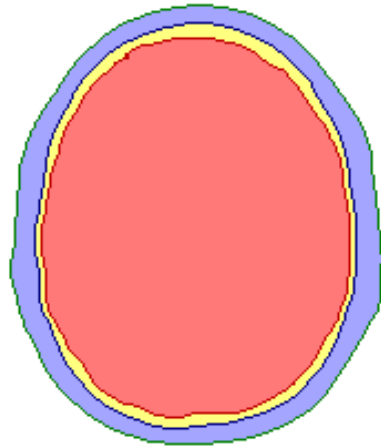
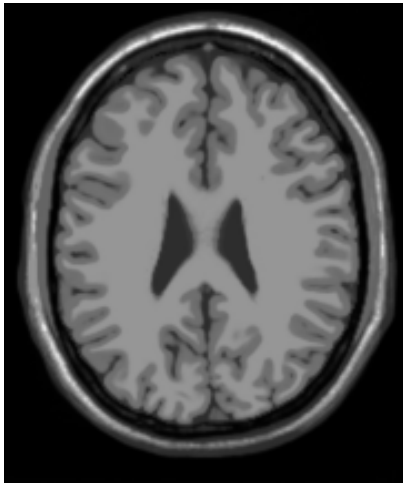
- Important tissues
 - skin
 - skull
 - brain
 - (CSF)
- Each compartment is
 - homogenous
 - isotropic
- Triangulated surfaces describe boundaries



Volume conductor: Boundary Element Method



- Construction of geometry
 - segmentation in different tissue types
 - extract surface description
 - downsample to reasonable number of triangles





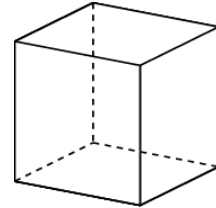
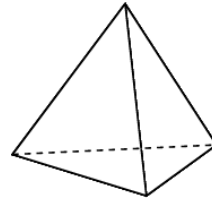
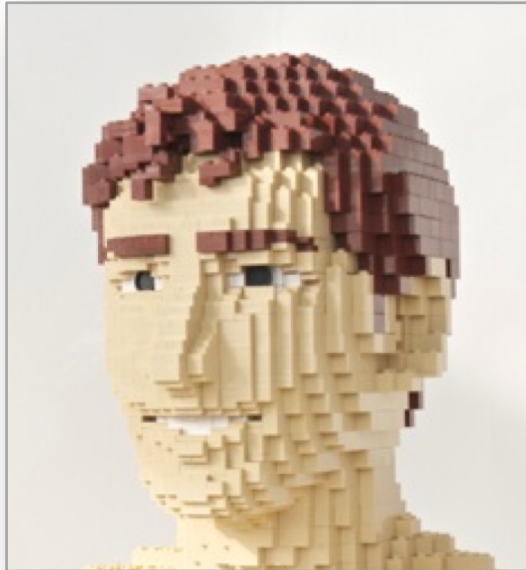
Volume conductor: Boundary Element Method

- Construction of geometry
 - segmentation in different tissue types
 - extract surface description
 - downsample to reasonable number of triangles
- Computation of model
 - independent of source model
 - only one lengthy computation
 - fast during application to real data
- Can also include more complex geometrical details
 - ventricles
 - holes in skull
 - dura

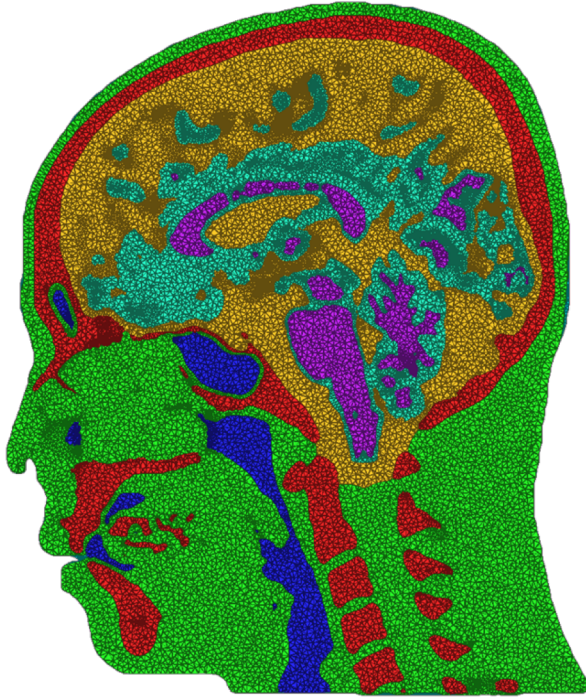
Volume conductor: Finite Element Method



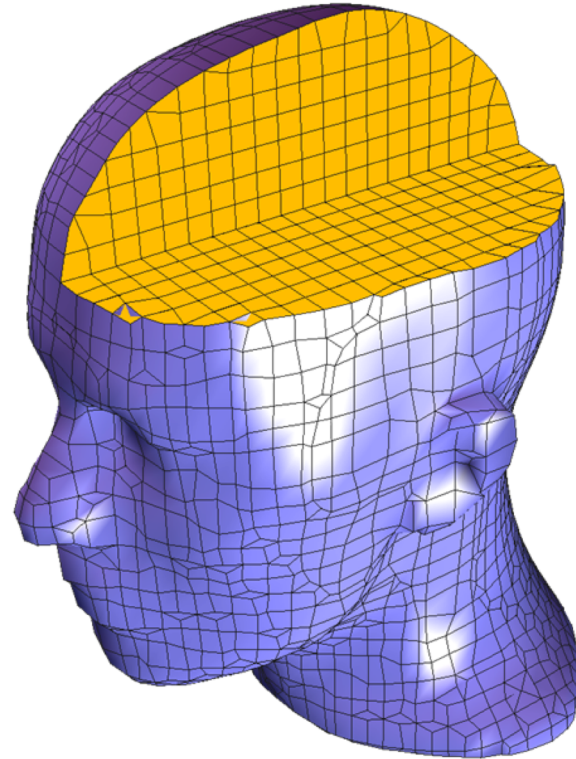
- Tessellation of 3D volume in tetrahedra or hexahedra



Volume conductor: Finite Element Method



tetraeders

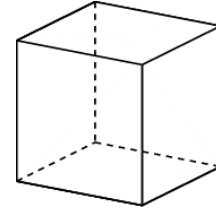
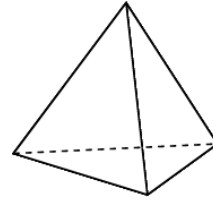


hexaheders



Volume conductor: Finite Element Method

- Tessellation of 3D volume in tetrahedra or hexahedra

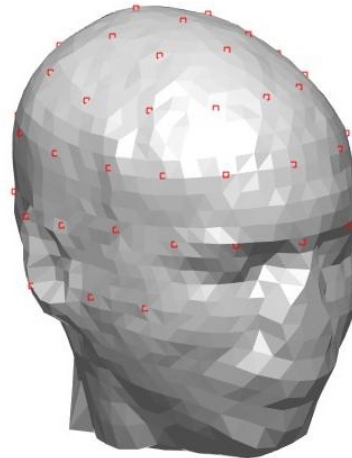
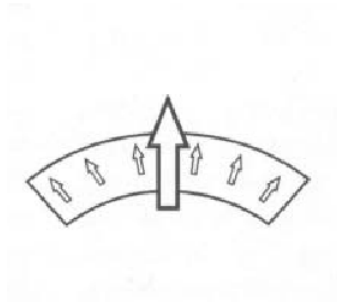
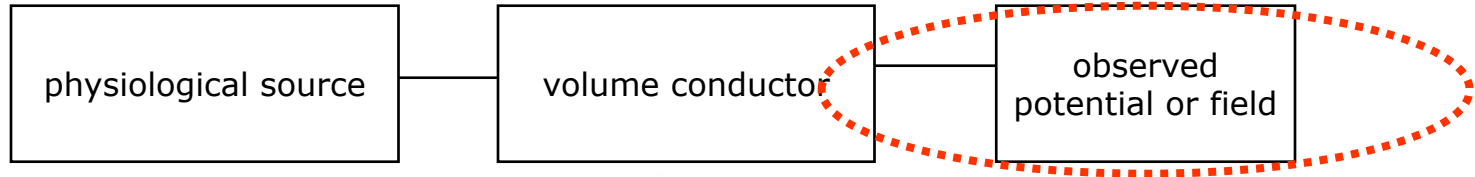


- Each element can have its own conductivity
- FEM is the most accurate numerical method but computationally quite expensive
- Geometrical processing not as simple as BEM

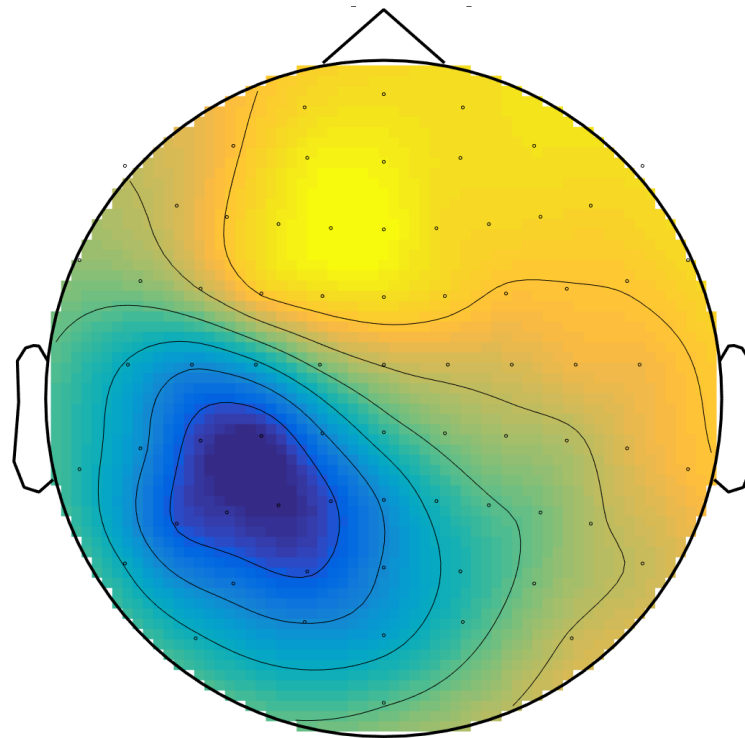
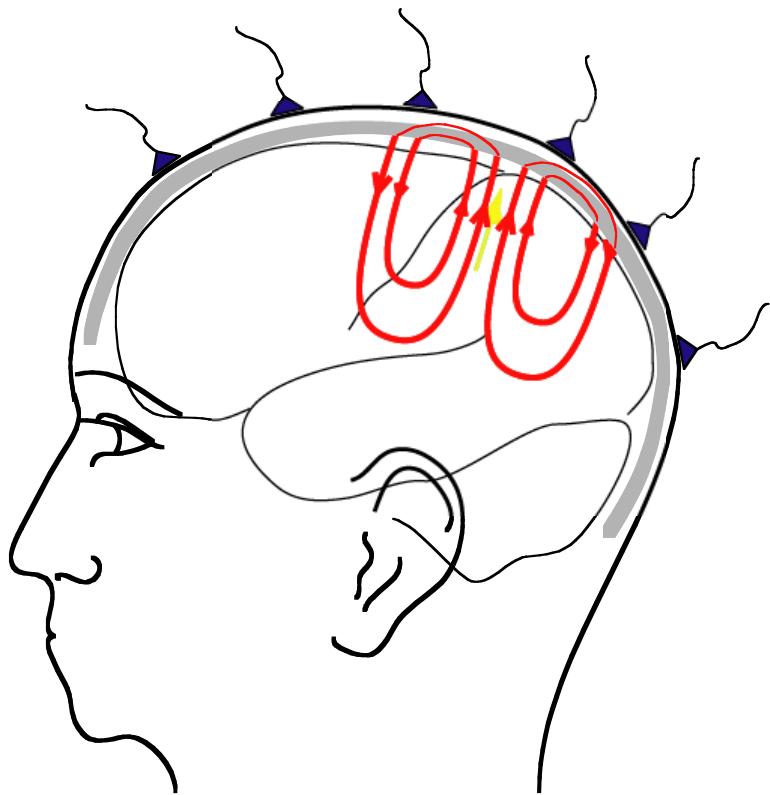
Biophysical source modelling: overview



forward model



EEG volume conduction





EEG volume conduction

- Potential difference between electrodes corresponds to current flowing through skin
- Only tiny fraction of current passes through skull
- Therefore the model should describe the skull and skin **as accurately as possible**



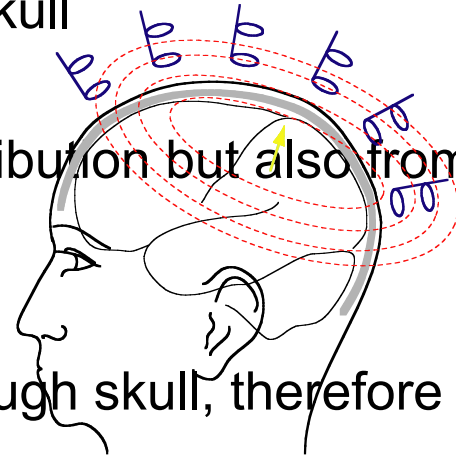
MEG volume conduction

- MEG measures magnetic field over the scalp

- Magnetic field is distorted by skull

- Magnetic field is not uniform distribution but also from the

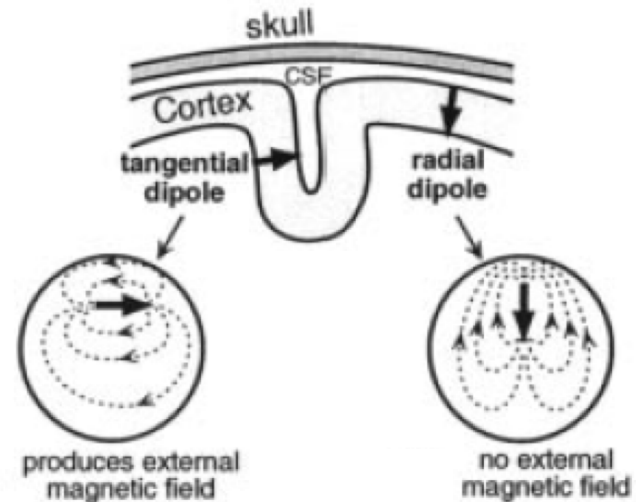
- Only tiny fraction of current flows through skull, therefore the model can ignore



MEG volume conduction compared to EEG



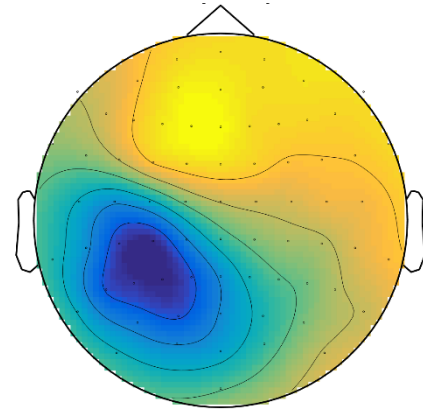
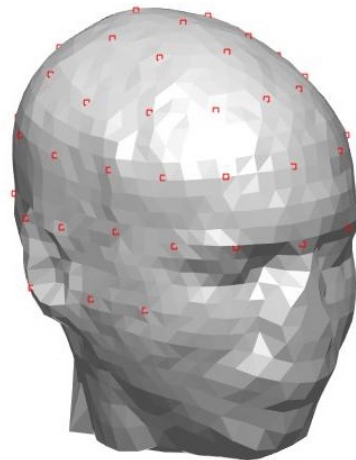
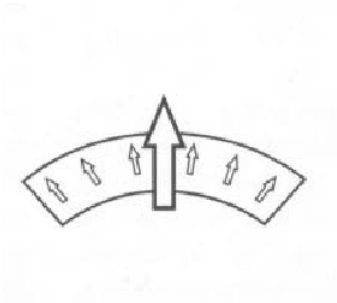
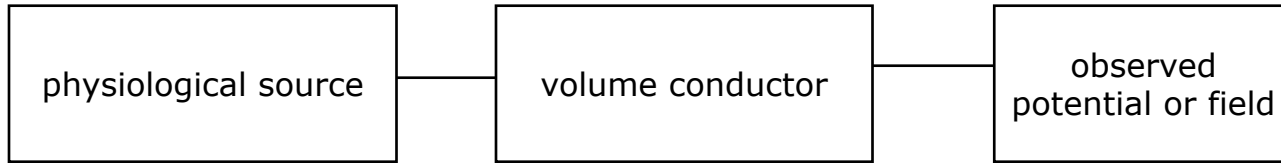
- EEG is measurement on scalp
 - potential difference due to volume currents
- MEG field not affected by head
 - magnetic field due to primary current (source)
 - magnetic field due to secondary (volume) currents



Biophysical source modelling: overview



~~forward model~~
inverse model





Motivation and background

Forward modeling

Source model

Volume conductor model

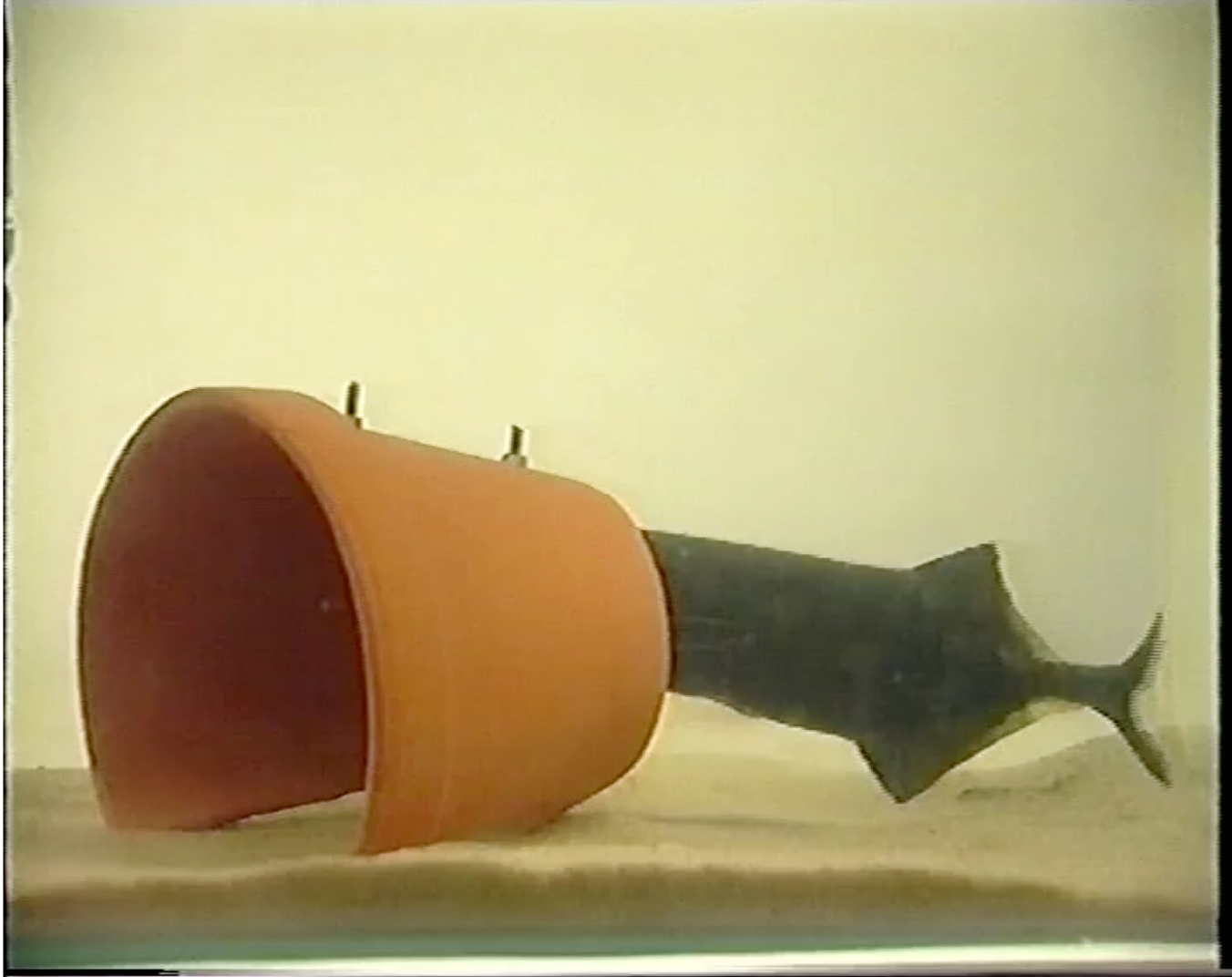
Inverse modeling

Single and multiple dipole fitting

Distributed source models

Beamforming methods

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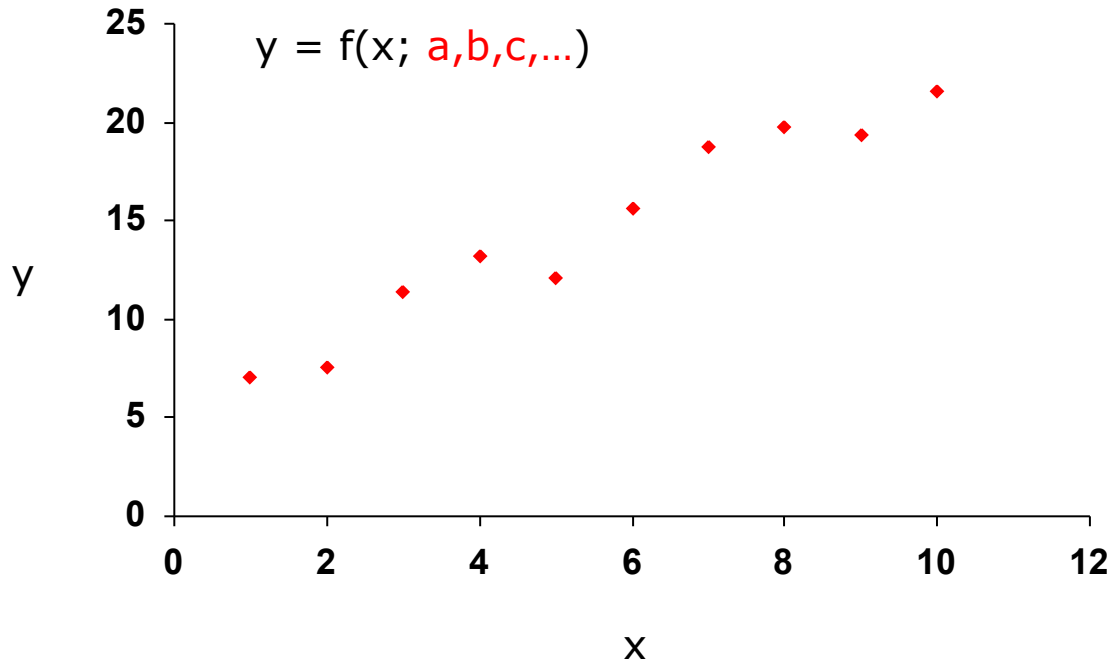
Summary



Inverse methods

- Single and multiple dipole models
 - Few patches of cortical activity
- Distributed source models
 - Distributed activity over the whole cortex
- Spatial filtering
 - Activity of different sources is uncorrelated to each other and to the noise

Single or multiple dipole models – Parameter estimation



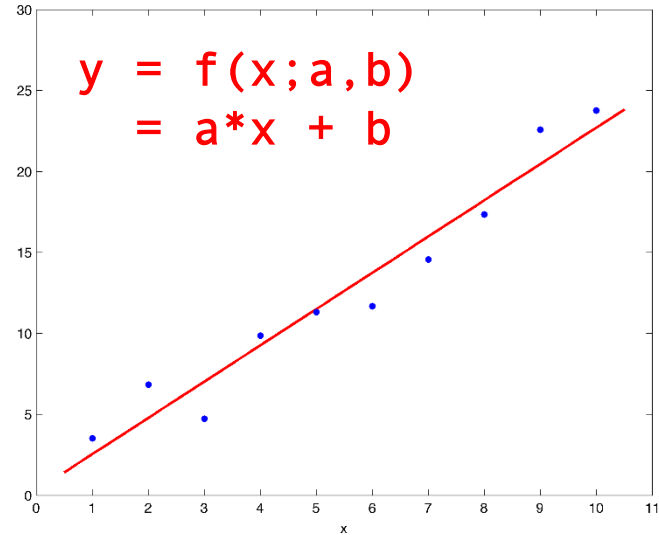
Parameter estimation: dipole parameters



source model with
few parameters

position
orientation
strength

minimize difference
between actual and
model data



Linear parameters: superposition of sources



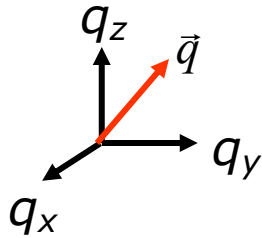
- three sources with parameters ζ_1 , ζ_2 and ζ_3

$$\left. \begin{array}{l} Y(\xi_1) \\ Y(\xi_2) \\ Y(\xi_3) \end{array} \right\} Y_{combined} = Y(\xi_1) + Y(\xi_2) + Y(\xi_3)$$

Linear parameters: estimation



$$Y = G_x q_x + G_y q_y + G_z q_z = \begin{bmatrix} G_{x,1} & G_{y,1} & G_{z,1} \\ G_{x,2} & G_{y,2} & G_{z,2} \\ \vdots & \vdots & \vdots \\ G_{x,N} & G_{y,N} & G_{z,N} \end{bmatrix} \cdot \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \mathbf{G} \cdot \vec{q}$$



$$Y = \mathbf{G} \cdot \vec{q}$$
$$= \mathbf{G}(\xi) \cdot \vec{q}$$

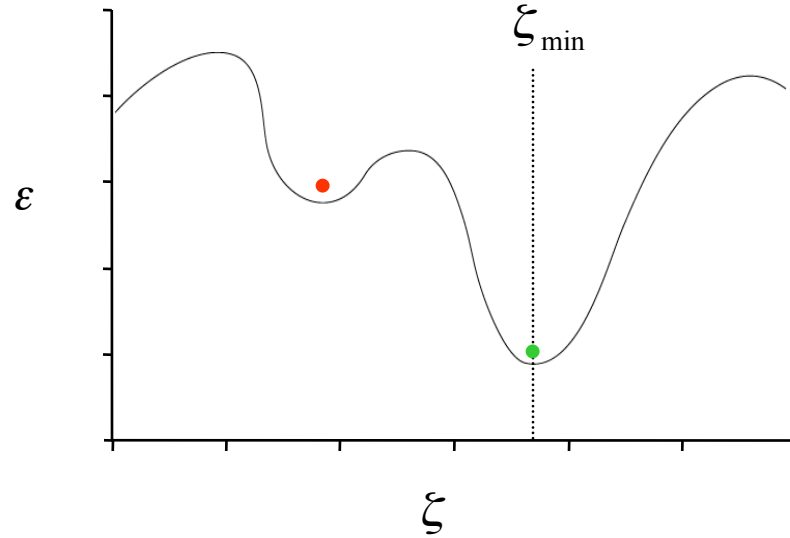
$$\vec{q} = \mathbf{G}^{-1} \cdot Y$$

Non-linear parameters



$$error(\xi) = \sum_{i=1}^N (Y_i(\xi) - V_i)^2 \Rightarrow \min_{\xi} (error(\xi))$$

$$\xi = a, b, c, \dots$$





Non-linear parameters: grid search

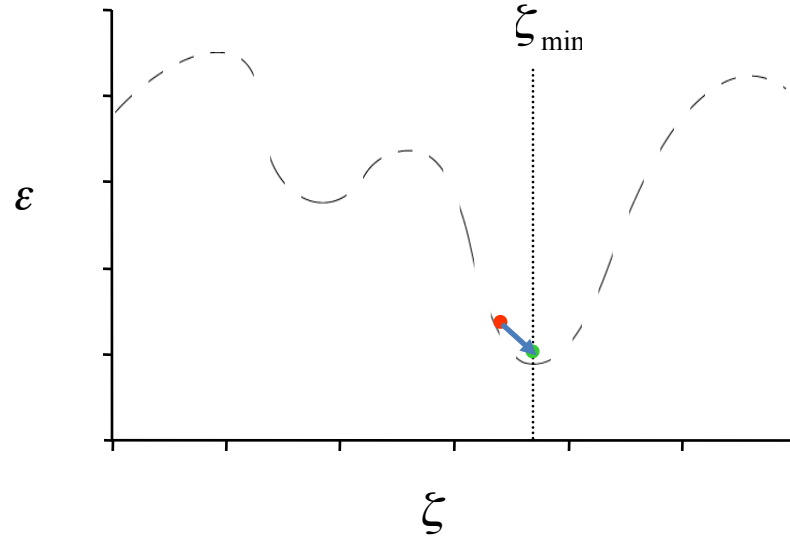
- One dimension, e.g. location along medial-lateral
 - 100 possible locations
- Two dimensions, e.g. med-lat + inf-sup
 - $100 \times 100 = 10.000$
- Three dimensions
 - $100 \times 100 \times 100 = 1.000.000 = 10^6$
- Two dipoles, each with three dimensions
 - $100 \times 100 \times 100 \times 100 \times 100 \times 100 = 10^{12}$



Non-linear parameters: Gradient descent method

$$error(\xi) = \sum_{i=1}^N (Y_i(\xi) - V_i)^2 \Rightarrow \min_{\xi} (error(\xi))$$

$$\xi = a, b, c, \dots$$

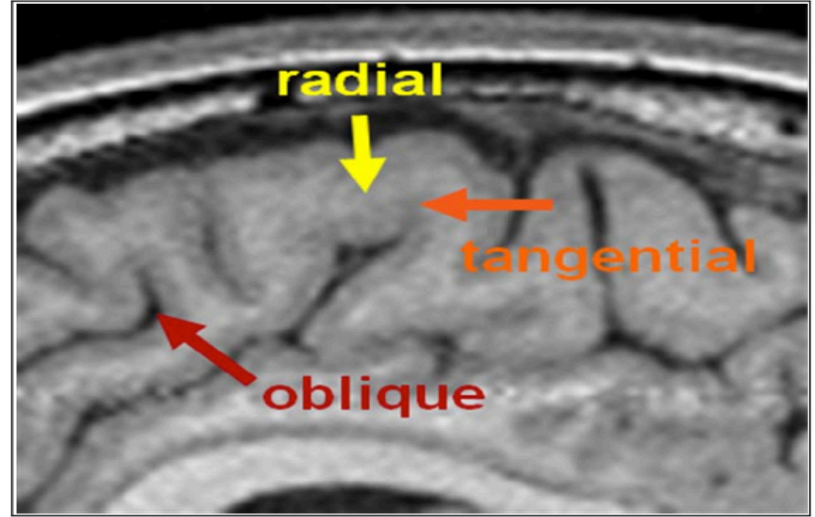
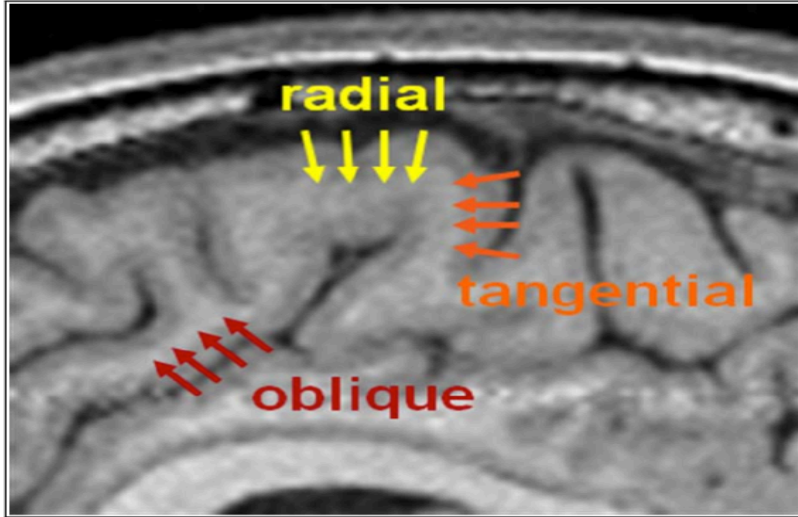




Single or multiple dipole models - Strategies

- Single dipole:
 - scan the whole brain, followed by iterative optimization
- Two dipoles:
 - scan with symmetric pair, use that as starting point for iterative optimization
- More dipoles:
 - sequential dipole fitting

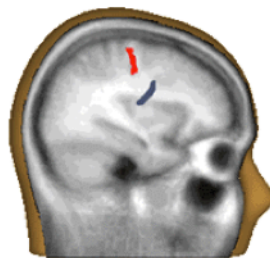
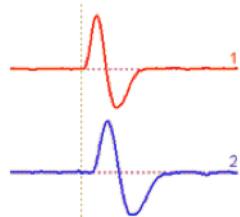
Sequential dipole fitting



Sequential dipole fitting



local currents

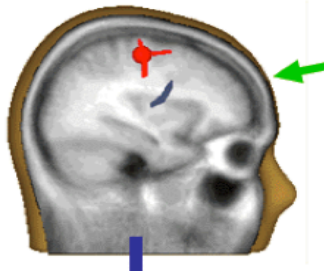


propagation

Sequential dipole fitting



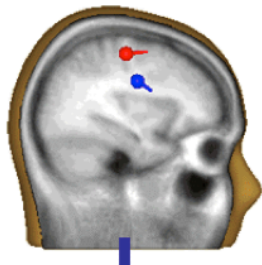
source model: 1



Sequential dipole fitting



source model: 4





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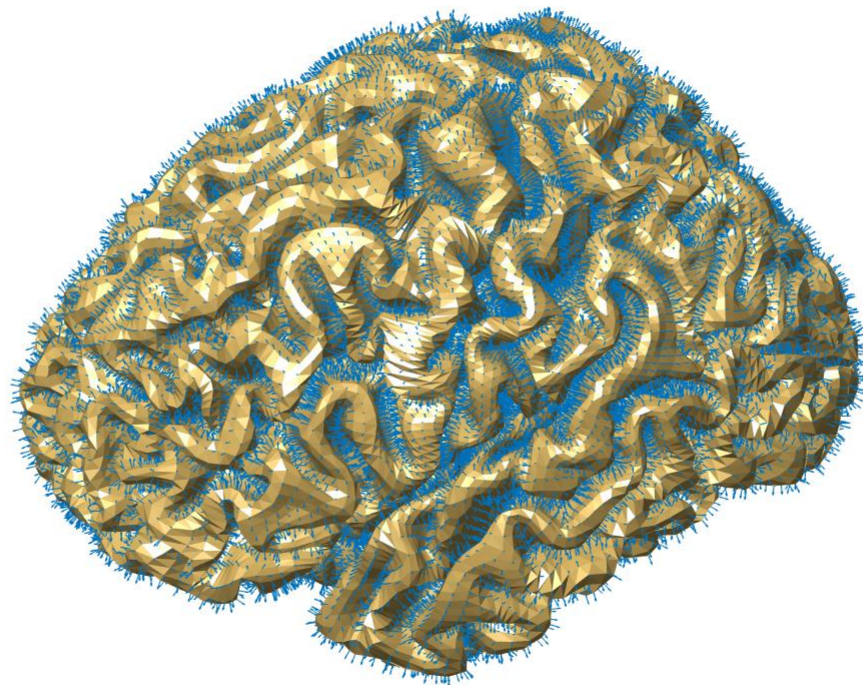
Summary

Distributed source model

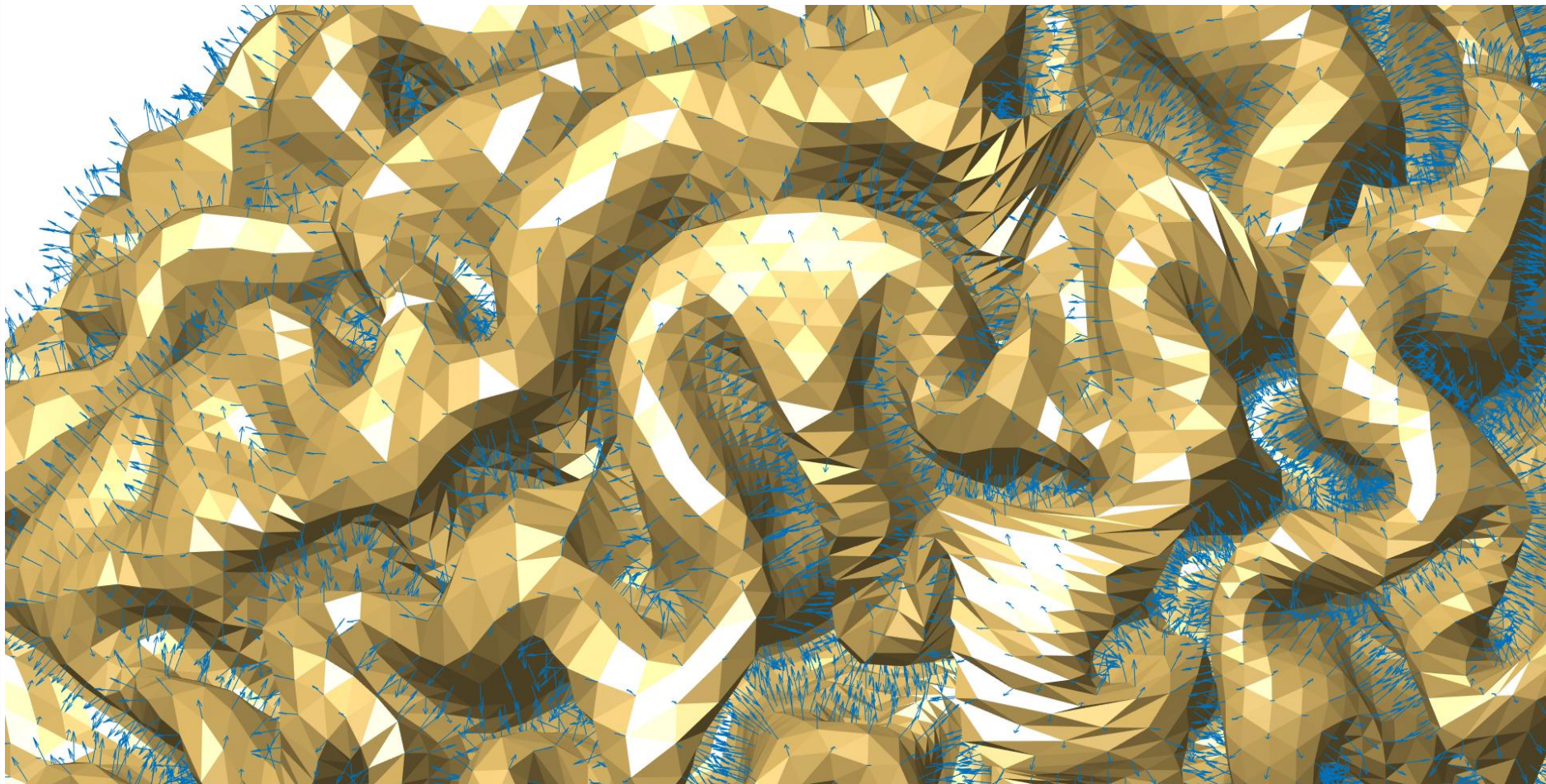


- Position of the source is **not estimated** as such
 - Pre-defined grid (3D volume or on cortical sheet)
- Strength is estimated
 - In principle easy to solve, however...
 - More “unknowns” (parameters) than “knowns” (measurements)
 - Infinite number of solutions can explain the data perfectly
 - Additional constraints required
 - Linear estimation problem

Distributed source model



Distributed source model



Distributed source model: linear estimation



$$Y = G_1 q_1 + G_2 q_2 + \dots = \begin{bmatrix} G_{1,1} & G_{2,1} & \dots \\ G_{1,2} & G_{2,2} & \dots \\ \vdots & \vdots & \ddots \\ G_{1,N} & G_{2,N} & \dots \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} = \mathbf{G} \cdot \vec{q}$$

$$\vec{q} = \mathbf{G}^{-1} \cdot Y$$

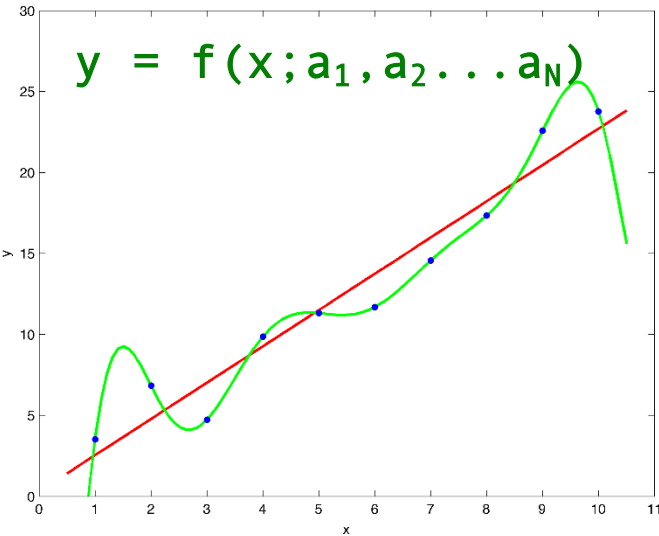
Distributed source model: linear estimation



distributed source model
with **many dipoles**
throughout the whole brain

estimate the strength of all
dipoles

data and noise can be
perfectly explained



Distributed source model: regularization



$$V = G \cdot q + \text{Noise}$$

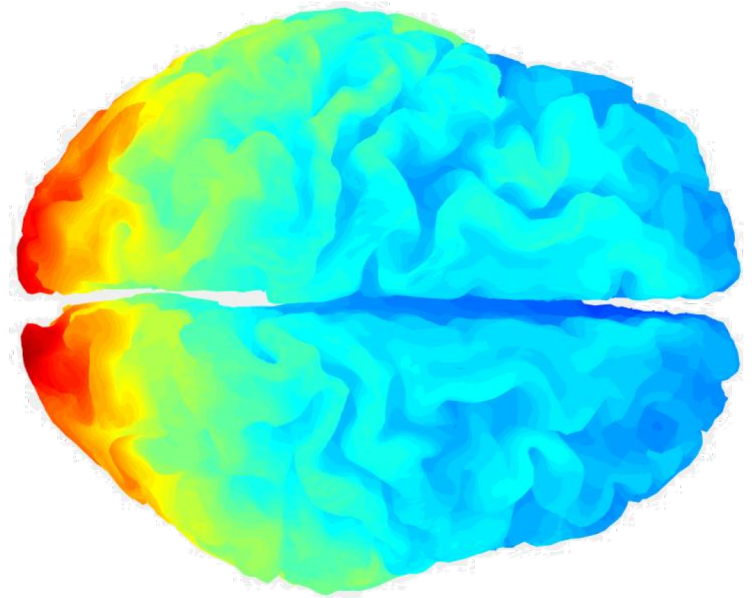
$$\min_q \{ \|V - G \cdot q\|^2 \} = 0 \quad !!$$

Regularized linear estimation:

$$\rightarrow \min_q \{ \underbrace{\|V - G \cdot q\|^2}_{\text{mismatch with data}} + \lambda \cdot \underbrace{\|D \cdot q\|^2}_{\text{mismatch with prior assumptions}} \}$$

mismatch with data

mismatch with prior
assumptions





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Spatial filtering with beamforming

- Position of the source is **not estimated** as such
- Manipulate filter properties, not source properties
 - No explicit assumptions about source constraints (implicit: single dipole)
 - Assumption that sources that contribute to the data should be uncorrelated

MORE TOMORROW!



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Summary 1



- Forward modelling
 - Required for the interpretation of scalp topographies
 - Different methods with varying accuracy
- Inverse modelling
 - Estimate source parameters from data
- Assumptions on source locations
 - Single or multiple point-like source
 - Distributed source
 - Spatial filtering

Summary 2



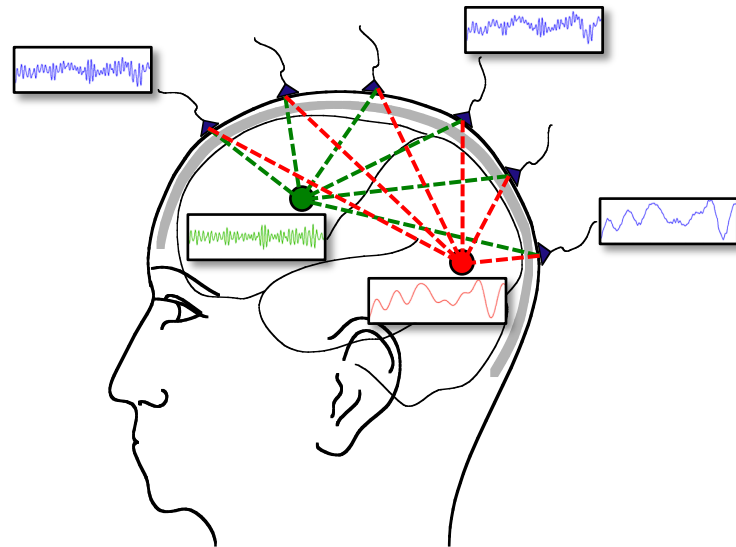
- Source analysis is not only about the “where” but also about untangling the “what” and “when”

spatial distribution of activity over the head
-> source reconstruction

timecourse of activity
-> ERP

spectral characteristics
-> power spectrum

temporal changes in power
-> time-frequency response (TFR)



Summary 3





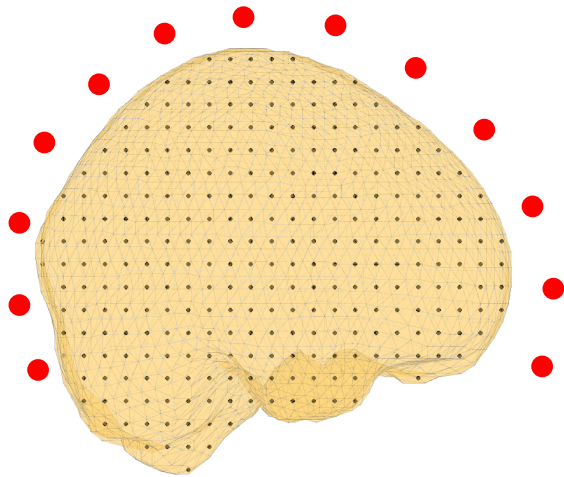
Forward and Inverse Modeling of EEG and MEG data





Forward and Inverse Modeling of EEG and MEG data Back-up

Single dipole solution with dipole fit



$$X_1 = 100$$

$$X_2 = 10$$

$$X_3 = 15$$

$$X_4 = 20$$

$$X_5 = 1$$

...

$$X_n = 3$$



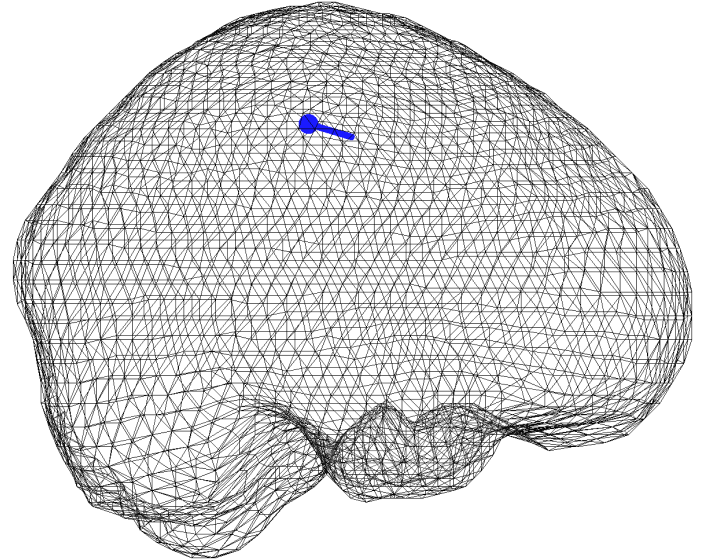
Further improvement
with
gradient descent

Single dipole solution with dipole fit



`ft_dipolefitting`

This uses the functional and anatomical data to perform the dipole fit



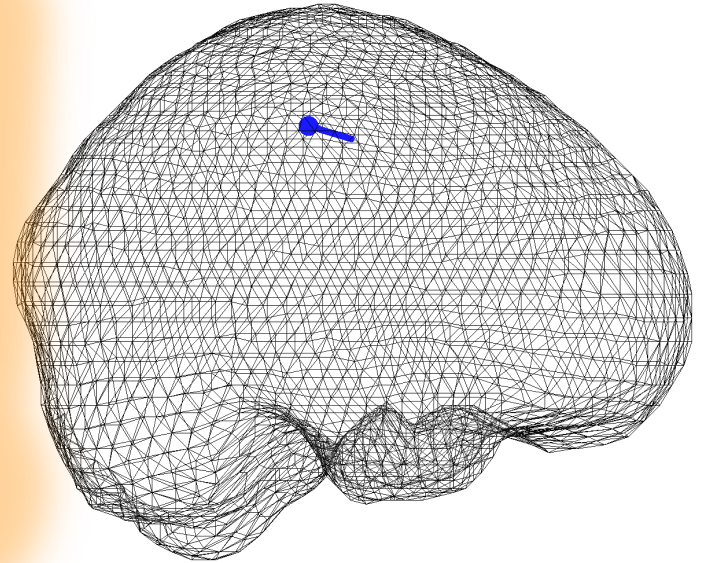
Single dipole solution with dipole fit



```
dipfit_bem.dip
```

```
ans =
```

```
pos: [13.958237048680118 34.388465910583285 97.809684095  
mom: [3x1 double]  
pot: [74x1 double]  
rv: 0.034549469532012  
unit: 'mm'
```

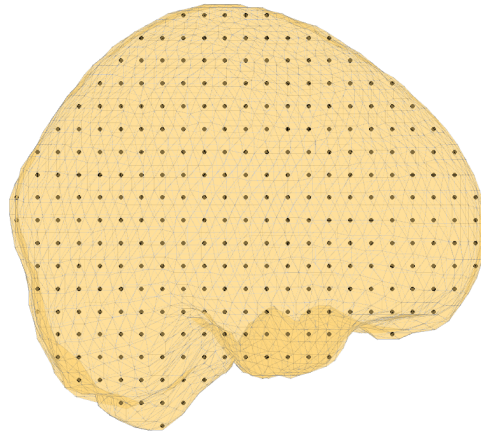


residual variance (rv) lies between 0 and 1

Distributed sources



- We are not looking for a **single** dipole location, but a **distributed** source



Distributed sources



Y is the measured data

A is our model

x is the distributed source model

λ is the regularisation parameter

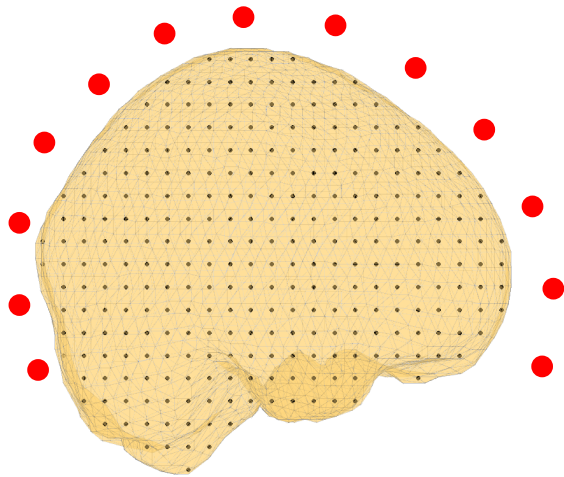
D is a operator

$$\min_x \{ \underbrace{\|Y - Ax\|_2}_{\text{mismatch with data}} + \underbrace{\lambda \|Dx\|_2}_{\text{mismatch with prior assumptions}} \}$$

mismatch with data

mismatch with prior
assumptions

Distributed sources



$$X_1 = 100$$

$$X_2 = 10$$

$$X_3 = 15$$

$$X_4 = 20$$

$$X_5 = 1$$

...

$$X_n = 3$$

All at the same time
+
regularization

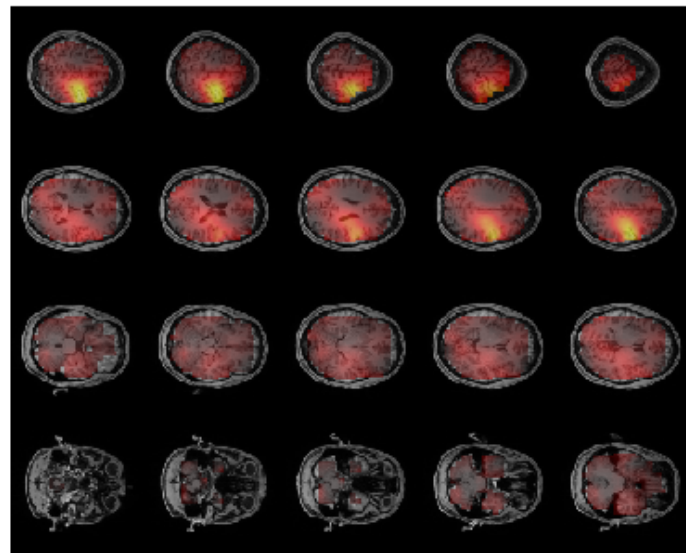
Single dipole solution with dipole fit



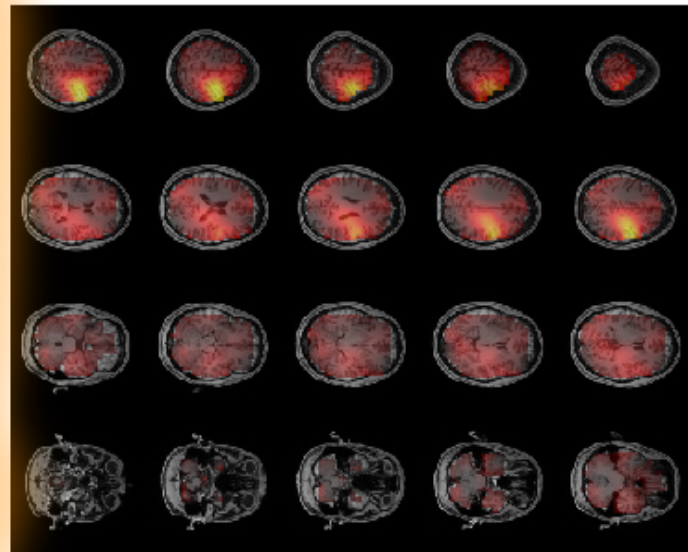
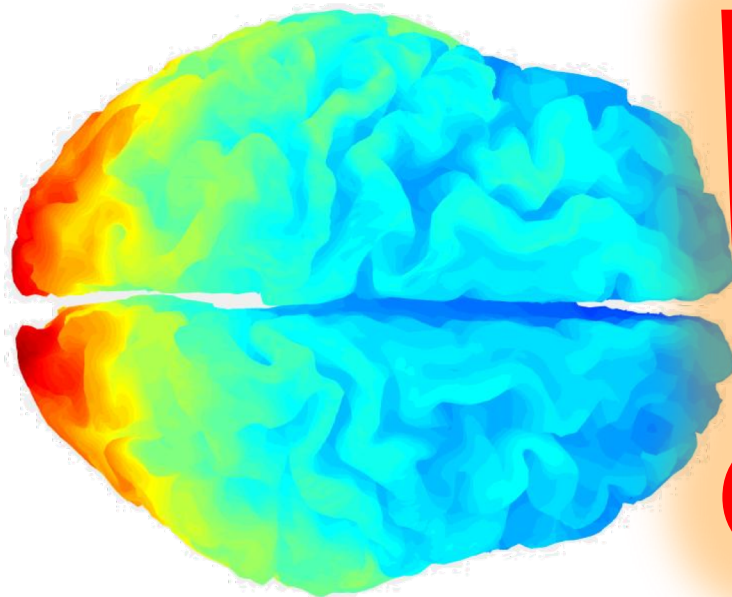
Cfg.mne. ...

`ft_dipolefitting(cfg,...)`

This uses the functional and anatomical data to perform mne



Distributed sources





Single and multiple dipole models

Minimize error between model and measured potential

Distributed source models

Perfect fit of model to the measured potential

Additional constraint on source smoothness, power or amplitude

Forward model

Influences the inverse solution

Take care of parameters



Forward and Inverse Modeling of EEG and MEG data

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Superposition of source activity

